Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate \( \iiint_E \sqrt{x^2 + y^2} \, dV \), where \( E \) is the solid between the elliptic paraboloids \( z = x^2 + y^2 \) and \( z = 18 - x^2 - y^2 \).

2. Express \( \iiint_E x \, dV \) as an iterated integral in cylindrical coordinates, where \( E \) is the solid that lies inside the cylinder \( x^2 + y^2 = 4 \) and inside the sphere \( x^2 + y^2 + z^2 = 16 \).

3. Convert \( \int_{-3}^3 \int_0^\sqrt{9-y^2} \int_0^\sqrt{9-x^2-y^2} z \sqrt{x^2 + y^2 + z^2} \, dz \, dx \, dy \) to cylindrical and spherical coordinates.

4. Using spherical coordinates, find the volume of the part of the ball \( \rho \leq 3 \) that lies between the cones \( \phi = \pi / 6 \) and \( \phi = \pi / 3 \).

5. Evaluate \( \iiint_E z^2 \, dV \), where \( E \) is bounded by the \( xy \)-plane and the hemispheres \( z = \sqrt{1 - x^2 - y^2} \) and \( z = \sqrt{9 - x^2 - y^2} \).

6. Use the transformation \( u = 3x, v = 2y \) to evaluate \( \iint_R e^{9x^2+4y^2} \, dA \), where \( R \) is the region in the first quadrant bounded by the ellipse \( 9x^2 + 4y^2 = 1 \).

7. Use the transformation \( x = 2u + v, y = u + 2v \) to express \( \iint_R (3x - y) \, dA \) as an iterated integral in the order \( dv \, du \), where \( R \) is the triangular region with vertices \((0,0), (2,1), \) and \((1,2)\).
SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 7 playlist (clickable link). You can turn on closed captions by clicking “CC” inside YouTube as well as adjust the video speed inside of “Settings” by clicking the cog in the bottom right of the player.

1. Evaluate $\iiint_E \sqrt{x^2 + y^2} \, dV$, where $E$ is the solid between the elliptic paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.
   $\frac{648\pi}{3}$

2. Express $\iiint_E x \, dV$ as an iterated integral in cylindrical coordinates, where $E$ is the solid that lies inside the cylinder $x^2 + y^2 = 4$ and inside the sphere $x^2 + y^2 + z^2 = 16$.
   $\int_0^{2\pi} \int_0^2 \int_{\sqrt{16-r^2}}^{\sqrt{16-r^2}} r^2 \cos \theta \, dz \, dr \, d\theta$

3. Convert $\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_0^\sqrt{9-x^2-y^2} z \sqrt{x^2 + y^2 + z^2} \, dz \, dx \, dy$ to cylindrical and spherical coordinates.
   Cyl: $\int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} \int_0^{\sqrt{9-r^2}} rz \sqrt{r^2 + z^2} \, dz \, d\theta$, \quad Sph: $\int_0^{\pi/2} \int_{-\pi/2}^{\pi/2} \int_0^3 \rho^4 \cos \phi \sin \phi \, d\rho \, d\theta \, d\phi$

4. Using spherical coordinates, find the volume of the part of the ball $\rho \leq 3$ that lies between the cones $\phi = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$.
   $9\pi \left( \sqrt{3} - 1 \right)$

5. Evaluate $\iiint_E z^2 \, dV$, where $E$ is bounded by the $xy$-plane and the hemispheres $z = \sqrt{1 - x^2 - y^2}$ and $z = \sqrt{9 - x^2 - y^2}$.
   $\frac{484\pi}{15}$

6. Use the transformation $u = 3x$, $v = 2y$ to evaluate $\int \int_R e^{9x^2+4y^2} \, dA$, where $R$ is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.
   $\frac{\pi}{24} (e - 1)$
7. Use the transformation \( x = 2u + v, \ y = u + 2v \) to express \( \iint_R (3x - y) \, dA \) as an iterated integral in the order \( dv \, du \), where \( R \) is the triangular region with vertices \((0, 0), (2, 1), \) and \((1, 2)\).

\[
\int_0^1 \int_0^{1-u} (15u + 3v) \, dv \, du
\]