Math 152 - Week-In-Review 7

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

   a. \( \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^{7/8}} \)
   
   b. \( \sum_{n=2}^{\infty} \frac{n \cos(\ln(n))}{2n^3 - 8} \)
   
   c. \( \sum_{n=1}^{\infty} \frac{(-1)^n e^{1/(n+1)}}{\sqrt[3]{n}} \)
   
   d. \( \sum_{n=1}^{\infty} \frac{(-3)^n - 1}{n^5} \)
   
   e. \( \sum_{n=1}^{\infty} \frac{(-5)^n n!}{3 \cdot 7 \cdot 11 \cdots (4n - 1)} \)

2. Find the interval and the radius of convergence for the following power series.

   a. \( \sum_{n=2}^{\infty} \frac{(-1)^n (x + 5)^n}{n6^n} \)
   
   b. \( \sum_{n=2}^{\infty} \frac{n!(7x - 3)^n}{9^n} \)
   
   c. \( \sum_{n=1}^{\infty} \frac{(-1)^n (2x - 1)^n}{\ln(n)} \)
Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 7 playlist (clickable link). You can turn on closed captions by clicking “CC” inside YouTube as well a adjust the video speed inside of “Settings” by clicking the cog in the bottom right of the player.

1. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

   a. \[ \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^{7/8}} \]
   - conditionally convergent

   b. \[ \sum_{n=2}^{\infty} \frac{n \cos(\ln(n))}{2n^3 - 8} \]
   - absolutely convergent

   c. \[ \sum_{n=1}^{\infty} \frac{(-1)^n e^{1/(n+1)}}{\sqrt[3]{n}} \]
   - conditionally convergent

   d. \[ \sum_{n=1}^{\infty} \frac{(-3)^{n-1}n^5}{4^{2n+1}} \]
   - absolutely convergent

   e. \[ \sum_{n=1}^{\infty} \frac{(-5)^{n}n!}{3 \cdot 7 \cdot 11 \cdots (4n - 1)} \]
   - divergent

2. Find the interval and the radius of convergence for the following power series.

   a. \[ \sum_{n=2}^{\infty} \frac{(-1)^n (x + 5)^n}{n^6} \]
   - \( R = 6, \ I = (-11, 1] \)

   b. \[ \sum_{n=2}^{\infty} \frac{n!(7x - 3)^n}{9^n} \]
   - \( R = 0, \ I = [3/7, 3/7] \)

   c. \[ \sum_{n=1}^{\infty} \frac{(-1)^n (2x - 1)^n}{\ln(n)} \]
   - \( R = 1/2, \ I = (0, 1] \)