

#### Problem 1:

# Write the null and alternative hypotheses in words and then symbols for each of the following situations.

1. New York is known as "the city that never sleeps". A random sample of 25 New Yorkers were asked how much sleep they get per night. Do these data provide convincing evidence that New Yorkers on average sleep less than 8 hours a night?

4.17 (a)  $H_0: \mu = 8$  (On average, New Yorkers sleep 8 hours a night.)  $H_A: \mu < 8$  (On average, New Yorkers sleep less than 8 hours a night.)

2. Employers at a firm are worried about the effect of March Madness, a basketball championship held each spring in the US, on employee productivity. They estimate that on a regular business day employee spend on average 15 minutes of company time checking personal email, making personal phone calls, etc. They also collect data on how much company time employees spend on such non- business activities during March Madness. They want to determine if these data provide convincing evidence that employee productivity decreases during March Madness.

(b)  $H_0: \mu = 15$  (The average amount of company time each employee spends not working is 15 minutes for March Madness.)  $H_A: \mu > 15$  (The average amount of company time each employee spends not working is greater than 15 minutes for March Madness.)

## Problem 2:

3. The nutrition label on a bag of potato chips says that a one ounce (28 gram) serving of potato chips has 130 calories and contains ten grams of fat, with three grams of saturated fat. A random sample of 35 bags yielded a sample mean of 134 calories, the population standard deviation is 17 calories. Is there evidence that the nutrition label does not provide an accurate measure of calories in the bags of potato chips? We have verified the independence, sample size, and skew conditions are satisfied



**4.23**  $H_0: \mu = 130$ .  $H_A: \mu \neq 130$ .  $Z = 1.39 \rightarrow$ p-value = 0.1646, which is larger than  $\alpha = 0.05$ . The data do not provide convincing evidence that the true average calorie content in bags of potato chips is different than 130 calories.

## Problem 3:

- 4. Suppose we had used a significance level of 0.01 in the sleep study. Would the evidence have been strong enough to reject the null hypothesis? (The p-value was 0.007.) What if the significance level was  $\alpha = 0.001$ ?
  - For an  $\alpha$ =0.01 the p-value= 0.007 <  $\alpha$ =0.01, therefore we have a strong enough evidence to reject the null hypothesis.
  - For an  $\alpha$ =0.001 the p-value= 0.007 >  $\alpha$ =0.01, therefore we don't have strong enough evidence to reject the null hypothesis.

## Problem 4

5. Suppose we are testing the hypotheses  $H_0: \mu = 100$  and  $H_0: \mu > 100$ . Which sample mean will have a smaller p-value  $\overline{X} = 110$  or  $\overline{X} = 115$ 

The one with the sample mean of 115 because it is further away from 100 towards the right tail. It will give more support to the alternative hypothesis.

## Problem 5

6. Suppose we had used a significance level of 0.01 in the sleep study. Would the evidence have been strong enough to reject the null hypothesis? (The p-value was 0.007.) What if the significance level was  $\alpha = 0.001$ ? For an  $\alpha = 0.01$  the p-value=  $0.007 < \alpha = 0.01$ , therefore we have a strong enough evidence to reject the null hypothesis. For an  $\alpha = 0.001$  the p-value=  $0.007 > \alpha = 0.01$ , therefore we don't have a strong enough evidence to reject the null hypothesis.

## Problem 6

7. Driver error can be listed as the cause of approximately 54% of all fatal auto accidents, according to the American Automobile Association. Thirty randomly selected fatal accidents are examined, and it is determined that 14 were caused by driver error.

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Using  $\alpha$ = 0.05, is the AAA proportion accurate? Run two-sided test to determine if the claimed value p = 0.54 is correct.

- a. No, we fail to reject the null hypothesis H0 : p = 0.54 at the  $\alpha = 0.05$  significance level.
- b. Yes, we fail to reject the null hypothesis H0 : p = 0.54 at the  $\alpha = 0.05$  significance level.
- c. No, we reject the null hypothesis H0 : p = 0.54 at the  $\alpha = 0.05$  significance level.
- d. Yes, we reject the null hypothesis H0 : p = 0.54 at the  $\alpha = 0.05$  significance level.

#### Problem 7

The university is interested in whether or not students support sport passes to be included in tuition and given to all students. This would raise tuition so it is controversial. They conduct a few samples and do a series of confidence intervals and hypothesis tests.

250 random students are sampled to estimate the proportion of students that support sports passes being included in tuition. Of those students 130 support it and 120 oppose.

8. What is the sample proportion and standard error of that proportion?

$$\hat{p} = \frac{133}{250} = 0.532$$
  $SE(\hat{p}) = \sqrt{\frac{0.532(1-0.532)}{250}}$ 

9. The university president wants to know if more than half of the students support sport passes being included in tuition. The sample proportion was .52. What would appropriate null and alternative hypothesis be?

e. H0: p = .5, Ha: p > .5 f. H0: p < .5, Ha: p = .5 g. H0: p = .52, Ha: p > .52 h. H0: p > .52, Ha: p = .52 i. H0:  $\hat{p}$  = .5 Ha:  $\hat{p}$  > .5 j. H0:  $\hat{p}$  > .5 Ha:  $\hat{p}$  = .5

- 10. The above hypothesis test is conducted. The p-value is 0.04 and the sample proportion was .52. What is the correct interpretation of this p-value
  - k. There is a 0.04 probability that the population proportion is .52.
  - I. The true proportion must be bigger than .50.
  - m. If the true proportion is 0.5, when multiple samples and hypothesis tests were done, approximately 4% of them would be have sample means further away from the null hypothesis.
  - n. There is a 0.04 probability that the null hypothesis is correct



# Problem 8

You wish to test a claim that  $\mu \neq 38$  at a level of significance of  $\propto = 0.05$ , and you are given the following information: x = 37.1, n = 35 and s = 2.7. The observations are plotted in the following histogram and qq plot.



11. Compute the value of the test statistic.

a) -3.12 b) -2.86 c) <mark>-1.97</mark> d) -1.83

$$t - test = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{37.1 - 38}{2.7/\sqrt{35}} = -1.97$$

12. Are the conditions satisfied for the reported test-statistic? *Yes, because the sample size is more than 30 observations and the t-test can be used.* 

## Problem 9

The hospital administrator randomly selected 64 patients and measured the time (in minutes) between when they checked in to the ER and the time they were first seen by a doctor. The average time is 137.5 minutes and the standard deviation is 39 minutes. She

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is getting grief from her supervisor on the basis that the wait times in the ER has increased greatly from last year's average of 127 minutes. However, she claims that the increase is probably just due to chance.

13. Are conditions for inference met? Note any assumptions you must make to proceed.

Independence: The sample is random and 64 patients would almost certainly make up less than 10% of the ER residents. The sample size is at least 30. No information is provided about the skew. In practice, we would ask to see the data to check this condition, but here we will make the assumption that the skew is not very strong.

14. Using a significance level of  $\alpha$ = 0.05, is the change in wait times statistically significant? Use a two-sided test since it seems the supervisor had to inspect the data before she suggested an increase occurred.

H0 :  $\mu = 127$ . HA :  $\mu \neq 127$ ; t-test = 2.15 ; p-value = 0.0316. Since the p-value is less than a= 0.05, we reject H0. The data provide convincing evidence that the average ER wait time has increased over the last year.

15. Would the conclusion of the hypothesis test change if the significance level was changed to  $\alpha$  = 0.01?

Yes, it would change. The p-value is greater than 0.01, meaning we would fail to reject H0 at  $\alpha =$  0.01.

## Problem 10

A patient named Diana was diagnosed with Fibromyalgia, a long-term syndrome of body pain, and was prescribed anti-depressants. Being the skeptic that she is, Diana didn't initially believe that anti-depressants would help her symptoms. However, after a couple months of being on the medication she decides that the anti-depressants are working, because she feels like her symptoms are in fact getting better.

16. Write the hypotheses in words for Diana's skeptical position when she started taking the anti-depressants.

HO: Anti-depressants do not help symptoms of Fibromyalgia. HA: Anti- depressants do treat symptoms of Fibromyalgia.

17. What is a Type 1 Error in this context?

Concluding that anti-depressants work for the treatment of Fibromyalgia symptoms when they actually do not.

18. What is a Type 2 Error in this context?

Concluding that anti-depressants do not work for the treatment of Fibromyalgia symptoms when they actually do.



#### Problem 11

Meteorologists in Texas want to increase the amount of rain delivered by thunderheads by seeding the clouds. Without a seeding, thunderheads produce, on average, 300 acre-feet. The meteorologists randomly selected 25 clouds which they seeded with silver iodide to test their theory that the average acre-feet is more than 300. The sample mean is 370.4 with a sample standard deviation of 300.1.



19. What conclusion can be drawn when the significance level is 0.05?

- o. The data does not provide statistical evidence that the average acre-feet from seeded clouds is more than 300.
- p. The data does provide statistical evidence that the average acre-feet from seeded clouds is more than 300.
- q. The data does not provide statistical evidence that the sample average acre-feet from seeded clouds is more than 300.
- r. Two of the above are correct.
- s. We cannot draw conclusions based on the p-value because the conditions are not met due to the extreme outliers.

#### Problem 12

The mean speed of internet in your apartment is usually 35 MBps. The internet provider, SuddenLink, charged you more for the last month. They claimed the mean speed of your internet connection to be more than 35 MBps. You are skeptical. So you tracked your daily internet speed for the last 30 days. Your speed data yields the sample mean 36.2 and the sample standard deviation 4.32.

20. Identify the appropriate null and alternative hypotheses.

H0 : μ = 35 vs Ha : μ > 35



- 21. After choosing the appropriate test (think of the appropriate null distribution carefully), what would be your conclusion at  $\alpha$  = 0.05 level of significance? We fail to reject the null hypothesis at the 5% level and conclude that SuddenLink was wrong.
- 22. Suppose the Sudden-Link manager asked you to provide a 95% confidence interval for the true mean speed of the Internet based on your sample.

