Math 308: Week-in-Review 6

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1. Find the general solution of the following equation

   (a)
   $$2y'' - 3y' + y = (t^2 + 1)e^t$$

   (b) Solve the initial value problem
   $$3y'' + 4y' + y = (\sin t)\, e^{-t}, \quad y(0) = 1, \;\; y'(0) = 0.$$

2. Find two linearly independent solutions of $t^2y'' - 2y = 0$ of the form $y(t) = t^r$. Using these solutions, find the general solution of $t^2y'' - 2y = t^2$.

3. One solution of $4t^2y'' + 4ty' + (16t^2 - 1)y = 0$, $t > 0$ is $y(t) = t^{-1/2}\cos(2t)$. Find the general solution of $4t^2y'' + 4ty' + (16t^2 - 1)y = 16t^{3/2}$.

4. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in, then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period, amplitude, and phase angle of the motion.

5. A spring is stretched 10 cm by a force of 3 N. A mass of 2kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is 5 m/s. If the mass is pulled down 5cm below its equilibrium position and given an initial velocity of 10 cm/s, determine its position u at any time. Find the quasifrequency of the motion.

6. A spring is stretched 6 in by a mass that weighs 8 lb. The mass is attached to a dashpot mechanism that has a damping constant of 0.25 lb s/ft. and is acted on by an external force of $4\cos(2t)$ lb.

   (a) Find the steady-state response of this system.

   (b) If the given mass is replaced by a mass m, determine the value of m for which the amplitude of the steady state response is maximum.

   (c) If the mass is the same as in the problem, determine the value of ω of the frequency of the external force $4\cos(\omega t)$ at which "practical resonance" occurs, i.e. the amplitude of the steady-state response is maximized.