Note ♯2: Exam 01 Review

Problem 1. (a) What is the radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0$?
(b) What is the intersection of the sphere with the xz-plane?

Problem 2. Find the scalar and vector projection of $\langle 12, 1, 2\rangle$ onto $\langle -1, 4, 8\rangle$.

Problem 3. Which of the following expressions are meaningful? Select all.
(a) $(\mathbf{a}\cdot\mathbf{b})\cdot\mathbf{c}$ (b) $\mathbf{a}\times(\mathbf{b}\cdot\mathbf{c})$ (c) $|\mathbf{a}|(\mathbf{b}\cdot\mathbf{c})$ (d) $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})$ (e) $(\mathbf{a}\cdot\mathbf{b})\times(\mathbf{c}\cdot\mathbf{d})$

Problem 4. Which of the following statements is correct?
(a) $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+\mathbf{k}$ are parallel
(b) $2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $-2\mathbf{i}+\mathbf{j}+2\mathbf{k}$ are orthogonal
(c) None of the above

Problem 5. Find the point at which the line $x = 2 - t, \quad y = 3t, \quad z = 1 + 2t$ intersects the plane $2x + 3y - z = 13$.

Problem 6. Are these skew lines(do not intersect and are not parallel)?

$$L_1: \quad x = 1 + 2t, \quad y = -2 - t, \quad z = 3 + 4t$$
$$L_2: \quad x = s, \quad y = 2 - s, \quad z = -3 - s$$

Problem 7. a) Find a scalar equation of the plane that passes through the points $P(2, 1, 3)$, $Q(3, -1, 2)$, and $R(4, 2, 4)$.
b) Find the area of the triangle determined by P, Q, R.

Problem 8. Find the equation of the following planes.
a) The plane passes through the point $(2, 1, -9)$ and is perpendicular to the line $x = 1 - 2t, y = -1 + 3t, z = 5t$.
b) The plane passes through the point $(3, 0, -4)$ and contains line $x = 1 + 2t, y = 2 - 3t, z = t$.
c) The plane passes through the point $(2, 1, -9)$ and is parallel to $6x + 5y = 3z + 5$.

Problem 9. Consider the planes $x + y + z = 2$ and $x + 2y + 2z = 1$.
a) Find the angle between the planes.
b) Find the line of intersection of these two planes.

Problem 10. Find the domain of the vector function $\mathbf{r}(t) = \left\langle \frac{t-3}{t-2}, \sin(\sqrt{t+3}), \ln(16 - t^2)\right\rangle$.

Problem 11. Find $\lim_{t\to 1}\mathbf{r}(t)$ where $\mathbf{r}(t) = \left\langle \frac{\sin(\pi t)}{\ln t}, \frac{t-1}{t^2+3t-4}, te^{-2t}\right\rangle$.

Problem 12. Given the curves $r_1(t) = \langle 1 - \cos t, t, 3 - t\rangle$ and $r_2(s) = \langle s^2, \sin(s), 3 + s\rangle$ intersect at the point $(0, 0, 3)$, find the angle of intersection of the two curves.

Problem 13. Find parametric equations for the tangent line to the space curve
$\mathbf{r}(t) = \left\langle 2t^2 + t + 1, \sqrt{9t+16}, e^{t^2-t}\right\rangle$ at the point $(1, 4, 1)$.

Problem 14. Find the unit tangent vector $\mathbf{T}(t)$ to the curve $\mathbf{r}(t) = \langle \sin(2t), -\cos(2t), 4t\rangle$ at the point $(0, 1, 2\pi)$.

Problem 15. Find the length of the curve $\mathbf{r}(t) = \left\langle 6t, t^2, \frac{1}{9}t^3\right\rangle, 0 \le t \le 1$.

Problem 16. Find the curvature, κ, of $\mathbf{r}(t) = \langle \cos t, \sin t, 0\rangle$.

Problem 17. Given the velocity vector $\mathbf{v}(t) = \langle te^{-t}, \sin(2t), 3t^2\rangle$ and $\mathbf{r}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$. Find the position vector, $\mathbf{r}(t)$, at time t.