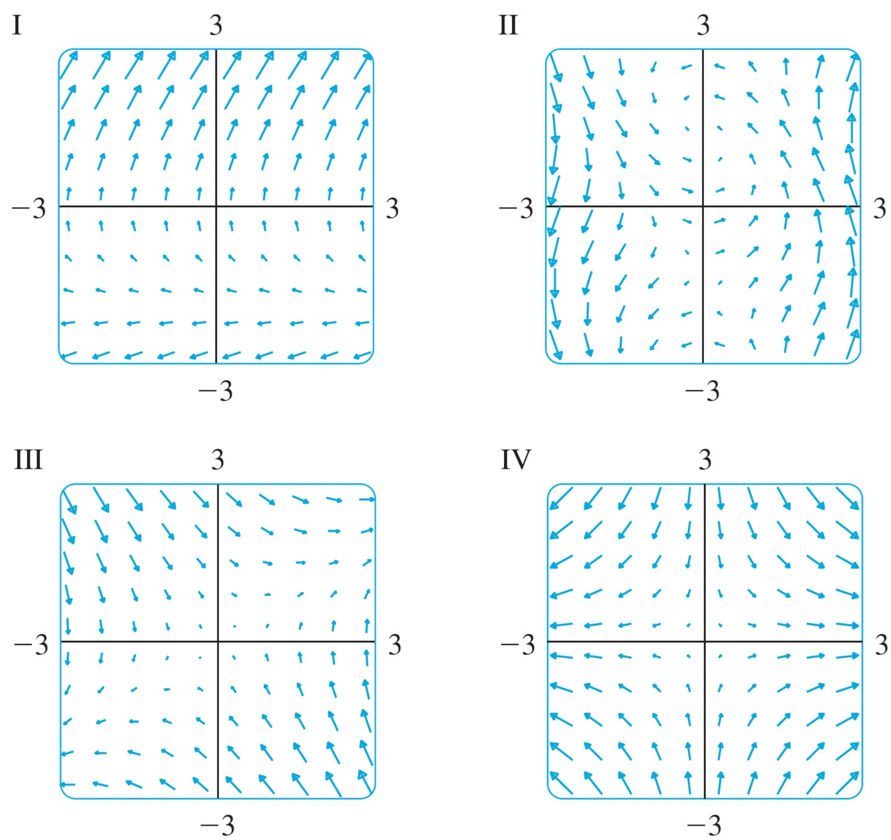
Note ♯8: Sections 16.1-16.3

Problem 1. Match the vector fields F with the plots labeled I-IV. Give reasons for your choices.
(a) $\mathbf{F}(x,y)=\langle x,-y\rangle$ (b) $\mathbf{F}(x,y)=\langle y,x-y\rangle$ (c) $\mathbf{F}(x,y)=\langle y,y+2\rangle$
(d) $\mathbf{F}(x,y)=\langle \cos(x+y),x\rangle$



Problem 2. Evaluate $\displaystyle\int_C xy^2ds$, where C is the right half of the circle $x^2+y^2=9$, oriented counterclockwise.

Problem 3. Evaluate $\displaystyle\int_C 2yds$, where C is the arc of the curve $x=y^2$ from $(1,-1)$ to $(4,2)$.

Problem 4. Evaluate $\displaystyle\int_C \left(x^2+y\right) ds$ where C consists of the line segment from the point $(1,4)$ to $(3,-1)$.

Problem 5. Evaluate $\displaystyle\int_C xyzds$, where C is the line segment from the point $(-2,0,3)$ to $(0,1,2)$.

Problem 6. Evaluate $\displaystyle\int_C ydx+x^2dy$, where C is decribed by $\mathbf{r}(t)=\left\langle 3e^t,e^{2t}\right\rangle, 0\leq t\leq 1$.

Problem 7. Evaluate $\displaystyle\int_C xdx+ydy$, where C is the arc of the parabola $x=4-y^2$ from $(-5,-3)$ to $(3,1)$.

Problem 8. Evaluate $\displaystyle\int_C (x+y)dz+(y-x)dy+zdx$ where $C: x=t^4, y=t^3, z=t^2, 0\leq t\leq 1$.

Problem 9. Evaluate $\int_C xydx+x^2dy+zdz$ where C is the line segment from $(0,-1,1)$ to $(2,3,-1)$.

Problem 10. Find the work done by the force field $\mathbf{F}(x,y) = \langle x^2, xy\rangle$ in moving an object counterclockwise around the right half of the circle $x^2 + y^2 = 9$.

Problem 11. Suppose we are moving a particle from the point $(0,0)$ to the point $(2,4)$ in a force field $\mathbf{F}(x,y) = \langle y^2, x\rangle$. Find $\int_C \mathbf{F}\cdot d\mathbf{r}$ where:

(a) The particle travels along the line segment from $(0,0)$ to $(2,4)$.

(b) The particle travels along the curve $y = x^2$ from $(0,0)$ to $(2,4)$.

Problem 12. Find $\int_C \mathbf{F}\cdot d\mathbf{r}$, $C : \mathbf{r}(\mathbf{t}) = \langle t, t^2, t^4\rangle, 0 \le t \le 1$, and $\mathbf{F}(x,y,z) = \langle x, z^2, -4y\rangle$.

Problem 13. Let $f(x,y) = 3x + x^2y - yx^2$. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = \nabla f$ and C is the curve given by $\mathbf{r}(t) = \langle 2t, t^2\rangle, 1 \le t \le 2$.

Problem 14. (a) Is $\mathbf{F}(x,y) = \langle 3x^2 - 4y, 4y^2 - 2x\rangle$ a conservative vector field? If so, find a function f so that $\mathbf{F} = \nabla f$.

(b) Is $\mathbf{F}(x,y) = \langle 2x + 4y, 4x - 1\rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

Problem 15. Given $\mathbf{F}(x,y) = \langle 2xy^3, 3x^2y^2\rangle$. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the curve given by $\mathbf{r}(t) = \langle t^3 + 2t^2 - t, 3t^4 - t^2\rangle, 0 \le t \le 2$.

Problem 16. Given that $\mathbf{F} = \langle 4xe^z, \cos(y), 2x^2e^z\rangle$ is conservative and $\mathbf{r}(t) = \langle \sin(t), t, \cos(t)\rangle$, compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $0 \le t \le \frac{\pi}{2}$. Note: We had to tell you $\mathbf{F}$ is conservative since we have not yet learned the testing criteria for conservativness in $\mathbb{R}^3$.