Math 150 - Week-In-Review 3

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Problem Statements

1. Write the given functions in standard form. Then determine the vertex, whether the vertex is a maximum or minimum, and the axis of symmetry.

   a) $g(x) = -3x^2 - 18x - 2$

   leading term $-3x^2$ $\Rightarrow$ $\cap$ $\Rightarrow$ vertex is maximum

   vertex $\left(-\frac{b}{2a}, g\left(-\frac{b}{2a}\right)\right)$

   $-\frac{b}{2a} = \frac{-(-18)}{2(-3)} = \frac{18}{2\times -3} = \frac{9}{-3} = -3$

   $g(-3) = -27 + 54 - 2 = 25$

   vertex $(-3, 25)$

   axis of symmetry $x = -2$

   $y = -3(x+3)^2 + 25$

   b) $f(x) = 4x^2 + 2x + 9$

   leading term $4x^2$ $\cup$ $\Rightarrow$ vertex is Min

   $-\frac{b}{2a} = \frac{-(2)}{2(4)} = -\frac{1}{4}$

   $f\left(-\frac{1}{4}\right) = 4\left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) + 9 = \frac{35}{4}$

   Vertex $\left(-\frac{1}{4}, \frac{35}{4}\right)$

   axis of sym. $x = -\frac{1}{4}$

   $y = 4\left(x + \frac{1}{4}\right)^2 + \frac{35}{4}$

2. Find the x-intercepts of the following functions.

   a) $h(x) = \frac{1}{3}x^2 - 4x + 3 = 0$

   $\times 3$ $\rightarrow$ $x^2 - 12x + 9 = 0$

   $x^2 - 12x + \left(\frac{-12}{2}\right)^2 - \left(\frac{-12}{2}\right)^2 + 9 = 0$

   $x^2 - 12x + (-6)^2 - 36 + 9 = 0$

   $(x-6)^2 - 27 = 0 \longrightarrow (x-6)^2 = 27$

   $x - 6 = \pm\sqrt{27} \longrightarrow x = 6 \pm \sqrt{27}$

b) $f(x) = 2x^{\frac{5}{2}} - x^{\frac{3}{2}} - x^{\frac{1}{2}}$

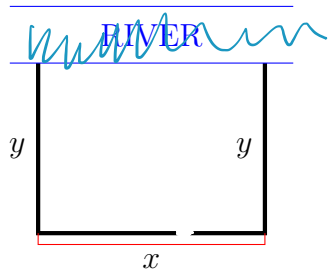
$= x^{\frac{1}{2}} \cdot \left[2x^2 - x - 1 \right] = 0 \rightarrow x^{\frac{1}{2}} = 0 \rightarrow \boxed{x = 0}$

$\hookrightarrow 2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$\boxed{x = 1}$ & $x = -\frac{1}{2}$ ✗ extraneous solution we can't have negative under even roots.

3. A farmer decides to enclose a rectangular stall against a river so his horses have water access. The figure below shows the shape he wants to make. If he has 1800 feet of fencing, what values for x and y will maximize the enclosed area with no fencing against the river? What is the maximum area he can enclose?



perimeter $2y + x = 1800$ ft

$y = -\frac{1}{2}x + \frac{1800}{2} = -\frac{1}{2}x + 900$

area $= x \cdot y$

$A = x\left(-\frac{1}{2}x + 900\right) = \underbrace{-\frac{1}{2}}_{a}x^2 + \underbrace{900}_{b}x + \underbrace{0}_{c}$

Vertex $\left(\frac{-900}{2(-\frac{1}{2})}, A(900)\right) = (900, 450)$

Max area : $x \times y = 405000$

Exam 1 Review

1. For the given polynomial functions, determine the end behavior of the graph.

a) $f(x) = 3x^8 + 7x^5 + 12$

$3x^8$ even power → parabola

↳ $a > 0$ ⇒ ↑ ↑

max # of turning points 7

12 y-int (0, 12)

b) $g(x) = -4x^9 - 3x^5 + 8$

leading Term

$-4x^9$ odd power → ↘ ↖

↳ $a < 0$ ⇒ decreasing

max # of turning points 8

2. Determine the quotient with fractional remainder (if necessary) of the following.

(a) $(7x^3 - 46x^2 - 14x + 3) \div (7x + 3)$ $= x^2 - 7x + 1$

$$\begin{array}{r|l} & x^2 - 7x + 1 \\ \hline 7x+3 & 7x^3 - 46x^2 - 14x + 3 \\ & -7x^3 + 3x^2 \\ \hline & -49x^2 - 14x + 3 \\ & -(-49x^2 - 21x) \\ \hline & 7x + 3 \\ & 7x + 3 \\ \hline & 0 \end{array}$$

(b) $(3x^3 - 2x^2 + 4x - 9) \div (x + 1)$ $= 3x^2 - 5x + 9 - \frac{18}{x+1}$

$$\begin{array}{r|l} & 3x^2 - 5x + 9 \\ \hline x+1 & 3x^3 - 2x^2 + 4x - 9 \\ & -3x^3 + 3x^2 \\ \hline & -5x^2 + 4x - 9 \\ & -(-5x^2 - 5x) \\ \hline & 9x - 9 \\ & -(9x + 9) \\ \hline & -18 \end{array}$$

3. Find the zeros and their multiplicities for the following functions, then determine the end behavior and maximum number of turning points. Roughly sketch the graph.

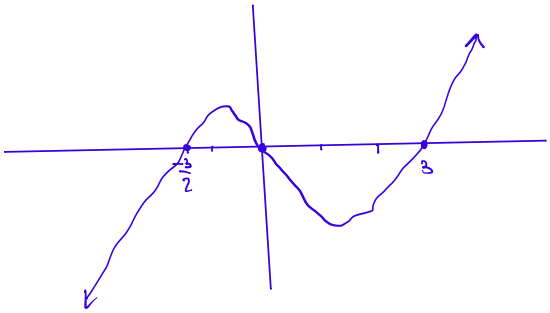
a) $k(x) = 2x^3 - 3x^2 - 9x$

$x(2x^2 - 3x - 9)$

$(2x+3)(x-3)$

$x = 0, -\frac{3}{2}, 3$ (all odd multiplicity $\Rightarrow$ passing through graph)

leading term $2x^3$ $\nearrow$ as $x \to \infty$



b) $g(x) = -x^4 + 8x^2 - 16$

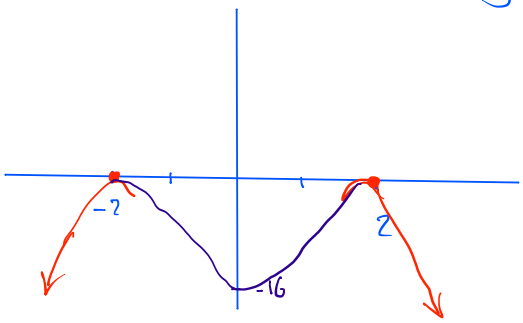
x-int: let $t = x^2$

$g(x) = -t^2 + 8t - 16$

$-(t^2 - 8t + 16) = 0$

$(t-4)^2 \Rightarrow t = 4$ with even multiplicity $\Rightarrow$ $t = x^2$ $\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$ w/ even multiplicity $\Rightarrow$ touch x-axis

y-int: $(0, -16)$



4. Let L_1 be the line passing through the points $(2, -1)$ and $(1, 5)$, and L_2 be the line passing through the points $(1, 4)$ and $(9, 8)$. Determine whether the lines are parallel, perpendicular, or neither.

L_1:

$$m_{L_1} = \frac{5-(-1)}{1-2} = \frac{6}{-1} = -6$$

$$m_{L_2} = \frac{8-4}{9-1} = \frac{4}{8} = \frac{1}{2}$$

Neither

5. Solve the inequality $|9 - 2x| - 2 > -1$.

$|9-2x| > -1 + 2$

$|9-2x| > 1 \iff 9-2x > 1$ or $9-2x < -1$

$-2x > 1-9$ $\quad -2x < -1-9$

$-2x > -8$ $\quad -2x < -10$

$x < 4$ or $x > 5$

6. Solve the quadratic equation $12x^2 + 12x = 3$ by completing the square.

$12x^2 + 12x - 3 = 0$

$12\left(x^2 + x - \frac{3}{12}\right) = 0$

$12\left(x^2 + x - \frac{1}{4}\right) = 0 \longrightarrow x^2 + x - \frac{1}{4} = 0$

$x^2 + x + \frac{1}{4} - \frac{1}{4} - \frac{1}{4} = 0$

$\left(x + \frac{1}{2}\right)^2 - \frac{1}{2} = 0$

$\left(x + \frac{1}{2}\right)^2 = \frac{1}{2}$

$x + \frac{1}{2} = \pm \frac{1}{\sqrt{2}}$

$x = -\frac{1}{2} \pm \frac{\sqrt{2}}{2}$

7. Solve the equation. Check for extraneous solutions.

$$\frac{12}{x^2 + 2x - 3} = \frac{3}{x - 1} + \frac{7}{x + 3}$$

→ from denominators $x \neq 1$, $x \neq -3$

$\frac{12}{x^2+2x-3} = \frac{3(x+3) + 7(x-1)}{(x-1)(x+3)}$

$\Rightarrow \frac{12}{x^2+2x-3} = \frac{3x+9+7x-7}{x^2+2x-3}$

$\Rightarrow 12 = 10x + 2$

$10x = 10$

$x = 1$ ⇒ not in domain of our equation

⇒ No solution

8. Given $f(x) = \sqrt{2x+1}$, $g(x) = \frac{1}{x}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and their domains.

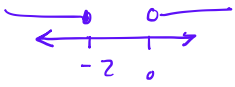
$2x+1 \geq 0 \quad x \geq -\frac{1}{2}$

$\text{Dom}(g) = (-\infty, 0) \cup (0, \infty)$

$\text{Dom } f = \left[-\frac{1}{2}, \infty\right)$

$f \circ g(x) = \sqrt{2(g(x))+1} = \sqrt{\frac{2}{x}+1} = \sqrt{\frac{2+x}{x}}$

$\frac{2+x}{x} \geq 0$ ($x=-2$, $x=0$)



$\text{Dom}(f \circ g) = (-\infty, -2] \cup (0, \infty)$

$g \circ f(x) = \frac{1}{f(x)} = \frac{1}{\sqrt{2x+1}}$ $\qquad \text{Dom}(g \circ f) = \left(-\frac{1}{2}, \infty\right)$

9. Find the intervals where the following inequality is true.

$$\frac{(11-x)^6(2x^2+5x-3)}{x+1} \geq 0$$

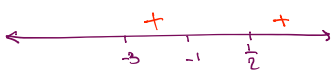
Note: $(11-x)^6$ is always positive because of the even power.

So the equation $x+1$ & $2x^2+5x-3$ are the ones that could possibly change the sign.

$\frac{+}{+}$ or $\frac{-}{-}$

Find critical points $x+1=0$ & $2x^2+5x-3=0$ $\quad (x+3)(x-\frac{1}{2})=0$

$x=-1$

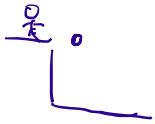


$[-3,-1) \cup [\frac{1}{2}, \infty)$

10. Antoine stands on a balcony and throws a ball to his dog, who is at ground level. The ball's height (in feet above the ground), t seconds after Antoine threw it, is modeled by $h(t) = -2t^2 + 4t + 16$

a) What is the height of the ball at the time it is thrown? b) What is the maximum height of the ball?

c) When does the ball reach it's maximum height?



a) if $t = 0 \rightarrow h(0) = 16$ ft.

b) (vertex $x = \frac{-b}{2a}$) $\quad t = \frac{-4}{-4} = +1 \quad \Rightarrow h(1) = 18$ ft max height of the ball

c) $\nearrow$ $t = 1$

11. Find the quadratic with axis of symmetry $x = 2$, a zero at (3,0), and a y-intercept of (0,16).

$y = a(x-h)^2 + k$ with (h,k) vertex

means vertex of $(2,k)$ $\Rightarrow y = a(x-2)^2 + k$

$0 = a(3-2)^2 + k$

$0 = a + k$

$16 = a(0-2)^2 + k$

$16 = 4a + k$

$$\begin{cases} a + k = 0 \rightarrow a = -k \\ 4a + k = 16 \end{cases} \rightarrow -4k + k = 16$$

$k = \frac{-16}{3}$ & $a = \frac{16}{3}$

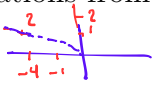
12. Consider the function $g(x) = -\frac{1}{3}\sqrt{-x+2} - 5$.

a) Identify the parent function f.

$f(x) = \sqrt{x}$

b) Describe the sequence of transformations from f to g.

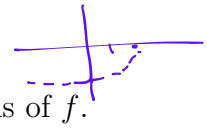
① $\sqrt{-x}$ reflect over y-axis



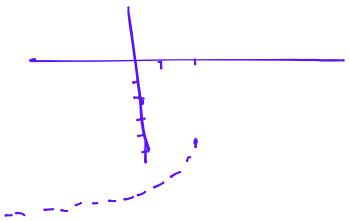
② $\sqrt{-(x-2)}$ right 2 units

③ $\frac{1}{3}\sqrt{-(x-2)}$ vertical shrink 3 units

④ $-\frac{1}{3}\sqrt{-(x-2)}$ reflect over x-axis



⑤ vertical shift 5 down



c) Use function notation to write g in terms of f.

$g(x) = -\frac{1}{3} f(-(x-2)) - 5$

d) Sketch the graph of g.