Math 308: Week-in-Review 8

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1. Express $f(t)$ in terms of the unit step function $u_c(t)$ and find it's Laplace transform

   (a)
$$f(t) = \begin{cases} (t-2)^2, & 0 \leq t < 2, \\ e^{t-2}, & t \geq 2. \end{cases}$$

(b)

$$f(t) = \begin{cases} 3, & 0 \leq t < 2, \\ 2t, & 2 \leq t < 4, \\ 3\sin(t-4), & t \geq 4. \end{cases}$$

2. Find the inverse Laplace transform of

(a)

$$F(s) = \frac{e^{-5s}}{s(s^4+4)}$$

(e)

$$F(s) = \frac{e^{-s} - e^{-4s}}{s(s^2+2s+5)}$$

3. Solve

   (a) $y'' + y = 3\sin(t) \cdot \delta(t - \pi/2),\ \ y(0) = 0,\ \ y'(0) = -1.$

(b) $y'' + 4y' = f(t),\ \ y(0) = y'(0) = 0$ where $f(t) = \begin{cases} 1, & 0 \le t < 1, \\ -1, & 1 \le t < 2, \\ 0, & t \ge 2. \end{cases}$

4. Let $f(t) = e^{-t}$ and $g(t) = \sin(t)$. Compute $(f * g)(t)$ and $(g * f)(t)$. Verify the Convolution Theorem for these functions.

5. (a) Find the Laplace transform of $h(t) = \int_0^t e^{t-x} \sin(x)\, dx$

(b) Find the inverse Laplace transform using the Convolution Theorem

$$F(s) = \frac{1}{(s^2+9)(s-2)}$$

6. Use Laplace transforms to solve the integro-differential equation

$$y' - 4y + 4\int_0^t y(x)\,dx = t^3 e^{2t}$$