Math 308: Week-in-Review 9

Shelvean Kapita

1. Find the following convolutions using the definition only

   (a) $e^t * e^{3t}$

   (b) $t * t^n$, where $n = 0, 1, 2, \cdots$

2. Using the Laplace transform (instead of the definition) compute the following convolutions

   (a) $u_a(t) * u_b(t)$

   (b) $t^n * t^m, \quad$ where $n = 0, 1, 2, \cdots$

3. In each of the following cases find a function (or generalized function) $g(t)$ that satisfies the equality for $t \geq 0$

   (a) $t * g(t) = t^4$

   (b) $1 * 1 * g(t) = t^2$

   (c) $1 * g(t) = 1$

4. Write the inverse Laplace transform in terms of a convolution integral

$$F(s) = \frac{s}{(s+1)^2(s+4)^3}$$

5. Solve the initial value problem

$$y'' - 2y' - 3y = g(t), \;\; y(0) = 1, \;\; y'(0) = -3.$$

6. Determine the radius of convergence for the power series

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}$

7. For the equation $(x^2+1)y'' + xy' - y = 0$

   (a) Determine a lower bound for the radius of convergence for the series solutions for the differential equation about $x_0 = 0$.

   (b) Seek its power series solution about $x_0 = 0$. Find the recurrence relation.

   (c) Find the general term of each solution $y_1(x)$ and $y_2(x)$

   (d) Find the first four terms in each of the solutions. Show that $W[y_1, y_2](0) \neq 0$.