



MATH 308: WEEK-IN-REVIEW 2  
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1. Find the general solution of the given differential equations

(a)  $y' + 2ty = 2te^{-t^2}$

(b)  $2\sqrt{xy}' = \sqrt{1 - y^2}$



(c)  $ty' + y = 3t \cos t, \quad t > 0$

2. Find the solution to the initial value problem and the interval of validity in each case

(a)  $2\sqrt{x} \frac{dy}{dx} = \cos^2(y), \quad y(4) = \frac{\pi}{4}$



$$(b) \frac{dy}{dt} + \frac{2y}{t} = \frac{\cos t}{t^2}, \quad y(1) = \frac{1}{2}, \quad t > 0$$



3. Find the value of  $y_0$  for which the solution of the initial value problem

$$y' - y = 1 + 2 \sin(t), \quad y(0) = y_0$$

remains finite as  $t \rightarrow \infty$



4. A 120 gallon tank initially contains 90 pounds of salt dissolved in 90 gallons of water. Brine containing 2 pounds per gallon of salt flows into the tank at a rate of 4 gallons per minute, and the well-stirred mixture flows out of the tank at a rate of 3 gallons per minute. How much salt does the tank contain when it is full?



5. Suppose you deposit \$6000 in an account that accrues interest continuously. Assuming no deposits and withdrawals, how much will be in the account after six years if the interest rate is a constant at 4.5% for the first three years and 5.4% for the last three years?



6. A population of rats at a corn farm increases at a rate proportional to their current population, and in the absence of other factors (no predators for example), the farmer notices that the rat population doubles in one month. There are 300 rats at the farm initially, and the farmer releases weasels that eat 100 rats per month.

(a) Determine the population of rats at the farm at any time.

(b) Find the time for the population of rats to double to 600 in the presence of weasels.



7. A ball with mass 1 kg is thrown upward with initial velocity 20 m/s from the roof of a building 50 m high. A force due to air resistance of  $v/10$  where the velocity is measured in m/s acts on the ball. Find the maximum height above the ground that the ball reaches.