Problem 1. Analyze the function $f(x) = (x^2 - 1)(x^2 - 7)$.

(1) What is the Domain of the function?

(2) What are the $x-$intercept(s) and what is the $y-$intercept of the function?

(3) Does the function have any vertical or horizontal asymptotes?

(4) What are the critical points of the function?

(5) On which interval(s) is the function increasing and decreasing?

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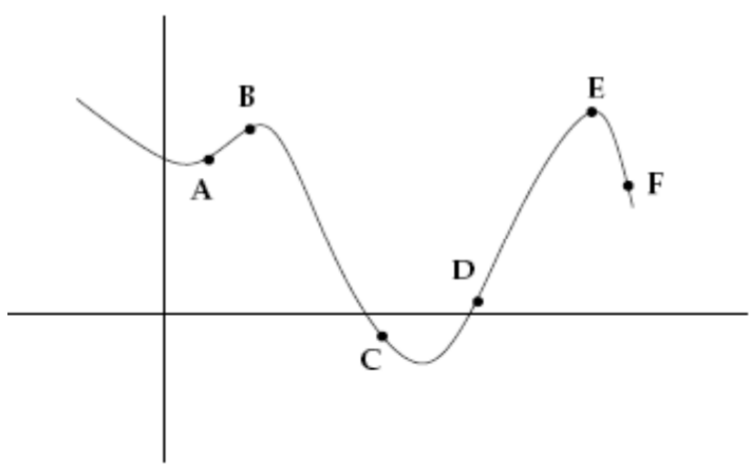
(6) Find the inflecton points of the function.

(7) On which interval(s) is the function concave up? Concave down?

(8) Sketch a graph of the function.

Problem 2. Discuss the relation between f, f' and f''.

Problem 3. At what points on the graph of $f(x)$ given below, does $f'(x)$ have the same sign as $f''(x)$? (Thanks to J. Kahlig)



Problem 4. Find the derivative of $f(x) = \left(\dfrac{6x}{5^x - 1}\right)^4$

Problem 5. Find the derivative of $f(x) = \left(4x - \sqrt{5 - x}\,\right)(\log_2 x)^3$

Problem 6. Find the value of a such that the tangent line to the curve $y = \ln x$ at $x = a$ passes through the origin.

Problem 7. A plane is flying directly away from you at 500 mph at an altitude of 3 miles. How fast is the distance between you and the plane increasing at the point when the horizontal distance between you and the plane is 4 miles?

Problem 8. You are inflating a spherical balloon at the rate of $7cm^3/s$. How fast is the radius of the balloon increasing when the radius is $4cm$?

Problem 9. A complany that makes deluxe toasters has a weekly demand equation given by $p(x) = 150e^{-0.02x}$, where $p(x)$ is the price per toaster in dollars, when x toasters are demanded.

(1) What is the marginal revenue for the company when 60 toasters are sold? Interpret your answer.

(2) At what value of x is the company's revenue increasing?

Problem 10. Sketch a graph of $f(x)$ given the following conditions:

a. Domain is $(-\infty, \infty)$

b. $f(-3) = -2, f(0) = 1, f(3) = -3, f(5) = -7, f(7) = -3$

c. $\lim_{x \to \infty} f(x) = 2$, $\lim_{x \to -\infty} f(x) = -6$

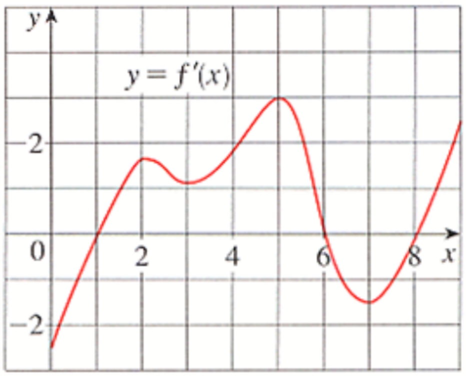
d. $f'(x) > 0$ on $(-\infty, 0) \cup (5, \infty)$

e. $f'(x) < 0$ on $(0, 5)$

f. $f''(x) > 0$ on $(-\infty, -3) \cup (3,7)$

g. $f''(x) < 0$ on $(-3, 3) \cup (7, \infty)$

Problem 11. Given the graph of $f'(x)$ below, find the intervals where $f(x)$



a) Is increasing

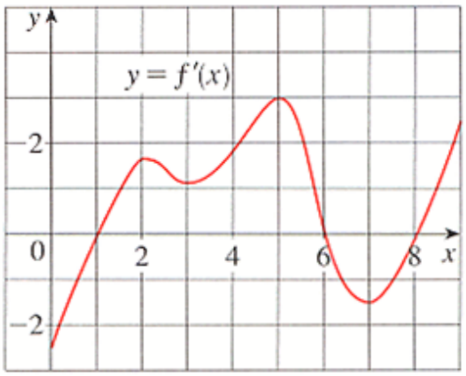
b) Is decreasing

c) Is concave up

d) Is concave down

e) Has maxima/minima/inflection pts

Problem 12. If the given graph is of $f''(x)$ instead, find the intervals where $f(x)$



a) Is increasing

b) Is decreasing

c) Is concave up

d) Is concave down

e) Has maxima/minima/inflection pts

Problem 13. The domain of $f(x)$ is $(-\infty, \infty)$ and $f(x)$ is twice differentiable on it's domain.

(1) If $f(5) = 10$, $f'(5) = 0$ and $f''(5) = -1$, what can we say about the function at $x = 5$?

(2) If $f(5) = -2$, $f'(5) = 0$ and $f''(5) = 3$, what can we say about the function at $x = 5$?

(3) If $f(5) = 10$, $f'(5) = 0$ and $f''(5) = 0$, what can we say about the function at $x = 5$?

(4) If $f(5) = -2$, $f'(5) = -1$ and $f''(5) = -2$, what can we say about the function at $x = 5$?

Problem 14. For a function $f(x)$, we are given that $f'(x) = \dfrac{-2x-8}{(x-1)^3}$ and $f''(x) = \dfrac{2(2x+13)}{(x-1)^4}$. The domain of $f(x)$ is given as $(-\infty, 1) \cup (1, \infty)$.

(1) What are the critical point(s) of the function $f(x)$?

(2) Use the Second Derivative Test to determine whether the critical point(s) are local maxima or local mimina of $f(x)$?

(3) What are the inflection point(s) of $f(x)$?

(4) What (if any) are the vertical and horizontal asymptotes of $f(x)$?

Problem 15. The price p, in dollars, and the demand x for a product are related by $2x^2+5xp+50p^2 = 80,000$ (Thanks to A.Allen)

(1) If the price is increasing at a rate of \$2 per month when the price is \$30, find the rate of change of demand with respevt to time.

(2) If the demand is decreasing at a rate of 6 units per month when the demand is 150 units, find the rate of change of price with respect to time.