Math 308: Week-in-Review 10

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Review for Exam 2

1. (3.5, 3.6) Find the general solution of the second order differential equation

   (a) $y'' + 4y' + 5y = e^{-2t} \sin t$

(b) (3.6) The general solution of the homogeneous equation $x^2y'' - 3xy' + 4y = 0$, $x > 0$, is given by $y_c(x) = c_1x^2 + c_2x^2 \ln x$. Find the general solution of the nonhomogeneous equation $x^2y'' - 3xy' + 4y = x^2 \ln x$, $x > 0$.

2. (3.7, 3.8) A string is stretched 10 cm by a force of 0.3 N. A mass of 0.25 kg is hung from the spring, and also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is 6 m/s. The mass is pulled down 5 cm below its equilibrium position and given an initial velocity of 10 cm/s downward.

   (a) Determine the position u as a function of time t

   (b) Find the quasifrequency of the motion.

   (c) If this system is also subjected to an external force $F(t) = 2\cos(4t)$, find $u(t)$, and the amplitude, period, and phase of the steady state motion.

3. (6.1) Find the Laplace transform of the following function using the definition of Laplace transform

(a) $f(t) = \begin{cases} t, & 0 \le t < 1, \\ 2 - t, & 1 \le t < 2, \\ 0, & t \ge 0 \end{cases}$

(b) Find the Laplace transform of the above function using Heaviside unit step functions.

4. (6.2, 6.3) Find the inverse Laplace transform of the function

$$F(s) = \frac{s^2 + s + 1}{(s^2 + 4)(s^2 - 9)}$$

5. (6.2, 6.3) Find the inverse Laplace transform of the function

$$F(s) = \frac{e^{-2s}(s^2 + s + 1)}{s(s+2)^2}$$

6. (6.3, 6.4) Find the solution of the initial value problem

(a) $y'' + 2y' + y = \begin{cases} \sin 2(t - \pi/2), & 0 \le t < \pi/2, \\ 0, & \pi/2 \le t < \infty \end{cases}, \quad y(0) = 1, \quad y'(0) = 0.$

(b) (6.5) $y'' + 2y' + y = e^{-t} + \delta(t-3),\ y(0) = 0,\ \ y'(0) = 3.$

7. (6.6)

(a) Use the definition of convolution to compute $(t * \sin t)$.

(b) Use the Convolution Theorem to find the inverse Laplace transform of

$$F(s) = \frac{s}{(s+1)(s^2+4)}$$

8. Find the radius and interval of convergence of the series

$$\sum_{n=0}^{\infty}(-1)^n \frac{2^n n!}{3 \cdot 5 \cdots (2n+1)} x^{2n+1}$$

9. (5.2) Consider the initial value value problem

$$y'' + x^2 y' + 2xy = 0, \;\; y(0) = 1, \;\; y'(0) = 0.$$

(a) Solve the initial value problem using a series of the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Find the recurrence relation.

(b) Find the first 6 terms of the series solution.

(c) Write down the solution using summation notation.