



## MATH 308: WEEK-IN-REVIEW 11

SHELVEAN KAPITA

1. Find the general solution of the system and the fundamental matrix. Classify the type of the critical point, and determine whether it is stable or unstable. Sketch the phase portrait.

(a)

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$$



(b)

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$$



(c)

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$$



2. Classify the types and stability of the equilibrium point(s) of the system

$$\mathbf{x}' = \begin{pmatrix} \alpha - 1 & \alpha + 1 \\ -2/3 & 1/3 \end{pmatrix} x$$

for different values of the parameter  $\alpha$ .



3. Find the general solution of the non-homogeneous system

$$\mathbf{x}' = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t^{-1} \\ 2t^{-1} + 4 \end{pmatrix}, \quad t > 0.$$



4. Find a particular solution of

$$\mathbf{x}' = \begin{pmatrix} e^{2t} & -1 \\ e^{4t} & e^{2t} \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ e^{2t} \end{pmatrix}$$

given the fundamental matrix

$$\mathbf{\Psi}(t) = \begin{pmatrix} e^{2t} & -1 \\ e^{4t} & e^{2t} \end{pmatrix}.$$



5. Apply the method of *undetermined coefficients* to find a particular solution of the non-homogeneous system

$$\mathbf{x}' = \begin{pmatrix} 7 & -4 \\ 2 & 3 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 9e^t \\ 25e^{-t} \end{pmatrix}$$