



Problem 1. A company that makes calculators has a cost function given by $C = 30x + 90,000$ dollars and a revenue function given by $R = 300x + \frac{x^2}{30}$ dollars, where x is the number of calculators produced and sold each week. If the number of calculators produced and sold is increasing at a rate of 500 calculators per week, find the rate of change of profit with respect to time when 6000 calculators are produced and sold each week.

$$x = 6000$$

rate \rightarrow wrt time $\rightarrow \frac{d?}{dt}$

Q: find $\frac{dP}{dt}$

= ?

$$P = R - C$$

$$= (300x - \frac{x^2}{30}) - (30x + 90,000)$$

$$= +300x - \frac{x^2}{30} - 30x - 90,000$$

$$\boxed{P = -\frac{x^2}{30} + 270x - 90,000}$$

$$\frac{dP}{dt} = \left(-\frac{1}{30}\right) \cdot 2x \cdot \frac{dx}{dt} + 270 \frac{dx}{dt} - 0$$

$$= -\frac{1}{30} \cdot 2 \cdot (6000) (500) + 270 (500)$$

$$\frac{dP}{dt} = -65,000 \text{ \$/week}$$

Problem 2. State the difference between partition numbers of f' and the critical values of f .

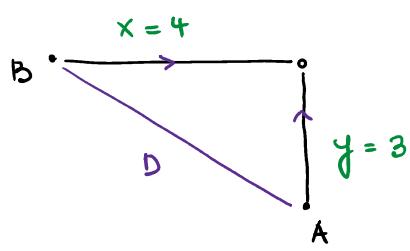
partition numbers of f' are x values when

- a) $f'(x) = 0$ or b) $f'(x)$ DNE

critical values of f are those x values from above which are in the domain of $f(x)$.

Problem 3. Two ships leave their respective ports at the same time and are headed to the same destination point. After one hour, ship A is 3 miles south of the destination port, traveling at 6 miles per hour. Ship B is 4 miles west of the destination port, traveling at 7 miles per hour. At what rate is the distance between the two ships changing?

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Find : $\frac{dD}{dt} = ?$

$$A: \frac{dy}{dt} = -6 \text{ miles/hr.}$$

$$B: \frac{dx}{dt} = -7 \text{ miles/hr.}$$

$$D = \sqrt{x^2 + y^2} \rightarrow \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

trick: $D^2 = x^2 + y^2$

$$2D \cdot \frac{dD}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$\frac{dD}{dt} = \frac{5}{2} [(4)(-7) + (3)(-6)]$$

$D=? \text{ when } x=4, y=3$

$$\frac{dD}{dt} = \frac{1}{5} (-28 - 18) = -\frac{46}{5}$$

$$= -9.2 \text{ miles per hour.}$$

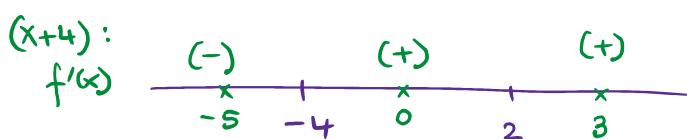
Distance between ships A & B is decreasing at a rate of 9.2 miles/hr.

Problem 4. Consider a function f that is continuous on its domain $D : (-\infty, 2) \cup (2, \infty)$ and where $f'(x) = \frac{8(x+4)}{(x-2)^4}$. Find all critical values of f , the intervals where f is increasing or decreasing, and where the local extrema of f occur. Label each type of extrema.

$$\begin{aligned} D: & (-\infty, 2) \cup (2, \infty) \rightarrow x \neq 2 \\ \Rightarrow f'(x) = & \frac{8(x+4)}{(x-2)^4} \rightarrow \text{partitions of } f'(x) \\ & \text{always } (+) \text{ve} \quad \text{a) } f'(x) = 0 \rightarrow 8(x+4) = 0 \\ & \quad \quad \quad \quad \quad \quad x+4 = 0 \Rightarrow x = -4 \\ & \text{b) } f'(x) \text{ DNE} \rightarrow (x-2)^4 = 0 \Rightarrow x \neq 2 \end{aligned}$$

critical values of $f(x)$: at $x = -4$

sign chart for $f'(x)$



$f(x)$ is decreasing on $(-\infty, -4)$

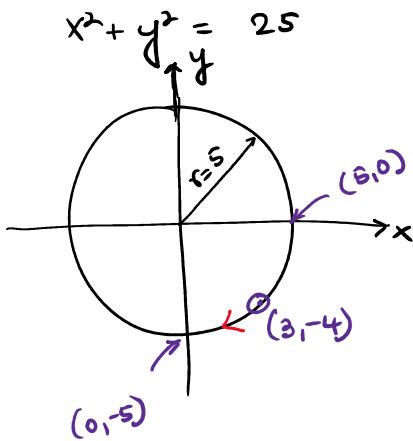
$f(x)$ is increasing on
 $(-4, 2) \cup (2, \infty)$



$f(x)$ has a local minima at $x = -4$

$x = 2$ is not in domain of $f(x)$

Problem 5. A car is moving along the graph of $x^2 + y^2 = 25$. When the car is at the point $\rightarrow (3, -4)$ on the graph, the x -coordinate is decreasing at a rate of 1.2 units per second. What is the rate of change of the y -coordinate at that time?



$$\boxed{\frac{dx}{dt} = -1.2}$$

find $\frac{dy}{dt} = ?$ |
 $x=3$
 $y=-4$

$$x^2 + y^2 = 25$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2y \frac{dy}{dt} = -2x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-x \frac{dx}{dt}}{y} = -\frac{x}{y} \cdot \frac{dx}{dt}$$

$$= -\frac{(3)}{(-4)} (-1.2)$$

$$= -0.9 \text{ units per second.}$$

ie y is also decreasing

Problem 6. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. If the radius is decreasing at a rate of 2.5 cms per second, what is the rate of change of the volume of the sphere, when the radius is 8 cms?

$$\text{Given: } V = \frac{4}{3}\pi r^3 \quad ; \quad \frac{dr}{dt} = -2.5 \text{ cm/sec}$$

$$\text{Find: } \frac{dv}{dt} \Big|_{r=8}$$

$$\frac{d}{dt}(V = \frac{4}{3}\pi r^3)$$

$$\frac{dv}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi (8)^2 \cdot (-2.5)$$

$$= -640\pi \text{ cm}^3/\text{sec}$$

$$= -2010.619 \text{ cm}^3/\text{sec}$$

Problem 7. Find the critical values of f , intervals where f is increasing or decreasing, and all local extrema of f . Classify each extrema.

$$(1) f(x) = \frac{3x^2 - 2x}{(x-4)^2}$$

Domain: $(-\infty, 4) \cup (4, \infty)$

$f(x)$ has a critical point
 $\textcircled{1} x = 8/22$

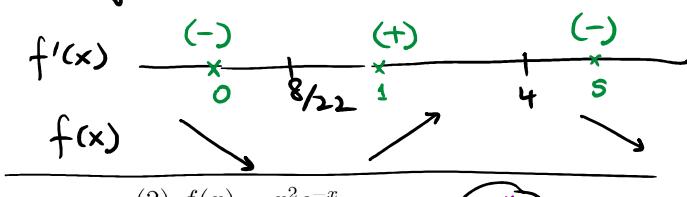
$$\begin{aligned} f'(x) &= \frac{(x-4)^2(6x-2) - (3x^2-2x)2(x-4)(1)}{((x-4)^2)^2} \\ &= \cancel{(x-4)} \left[(x-4)(6x-2) - 2(3x^2-2x) \right] \end{aligned}$$

$$f'(x) = \frac{6x^2 - 2x - 24x + 8 - 6x^2 + 4x}{(x-4)^3}$$

$$= \frac{-22x + 8}{(x-4)^3}$$

$$\begin{aligned} f'(x) &= 0 \\ -22x + 8 &= 0 \\ 22x &= 8 \\ x &= 8/22 \end{aligned}$$

Sign chart:



$$\begin{aligned} \textcircled{1} x=0, f'(x) &= \frac{8}{(-4)^3} \rightarrow f'(x) \text{ DNE} \\ \textcircled{2} x=1, f'(x) &= \frac{-14}{(-3)^3} = \frac{-14}{-27} = \frac{14}{27} \end{aligned}$$

$$\textcircled{3} x=5, f'(x) = \frac{-22(5)+8}{(5-4)^3} = \frac{-102+8}{1} = -\frac{94}{1}$$

Domain: $(-\infty, \infty)$

$$f'(x) = (2x)e^{-x} + (x^2)(e^{-x})(-1)$$

$$= 2xe^{-x} - x \cdot x \cdot e^{-x}$$

$$= xe^{-x}(2-x)$$

$$f'(x)=0 : \begin{array}{l} x=0 \\ \text{never zero} \end{array} \quad 2-x=0 \quad x=2$$

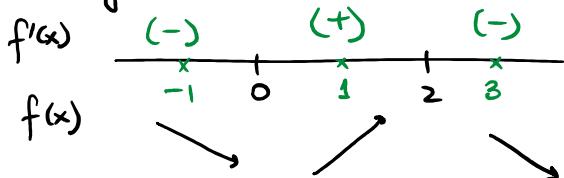
$f(x)$ is decreasing on $(-\infty, \frac{8}{22}) \cup (4, \infty)$

$f(x)$ is increasing on $(\frac{8}{22}, 4)$

$f(x)$ has a local minimum @ $x = \frac{8}{22}$

$f'(x)$ DNE \rightarrow never. $\rightarrow f(x)$ has critical points $\textcircled{1} x=0, x=2$

sign chart



$$f'(x) = xe^{-x}(2-x) \rightarrow$$

$$\textcircled{1} x=-1, f'(x) = (-1)(e^{-1})(2+1)$$

$$\textcircled{2} x=1, f'(x) = (1)(e^{-1})(1)$$

$$\textcircled{3} x=2, f'(x) = (2)(e^{-2})(-1)$$

$f(x)$ is decreasing on $(-\infty, 0) \cup (2, \infty)$

$f(x)$ is increasing on $(0, 2)$

$f(x)$ has a local minima @ $x=0$

$f(x)$ has a local maxima @ $x=2$

(3) $f(x) = 125 \ln(x) - \frac{5}{2}x^2$.

Domain: $(0, \infty)$

$$f'(x) = 125 \cdot \frac{1}{x} - \frac{5}{2} \cdot 2x = \boxed{\frac{125}{x} - 5x} \rightarrow f'(x)$$

$$f'(x)=0 \rightarrow \frac{125}{x} - 5x = 0 \Rightarrow \frac{125}{x} = 5x \Rightarrow 125 = 5x^2$$

$$x^2 = \frac{125}{5} = 25, \quad x = \pm 5$$

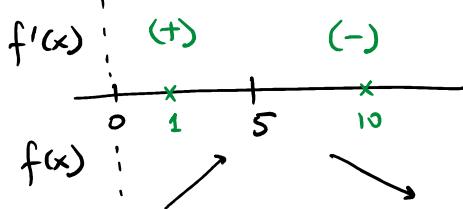
$f'(x)$ DNE @ $x=0$ → outside D.

But $x=-5$ is outside D.

$f(x)$ has ~ critical point @ $x=5$

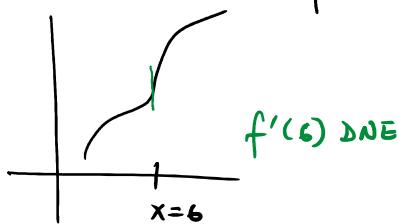
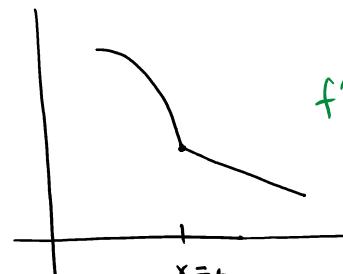
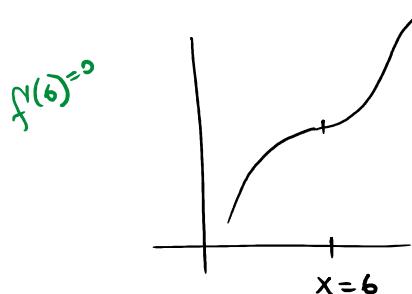
$f(x)$ increases on $(0, 5)$ & decreases on $(5, \infty)$

@ $x=5$, $f(x)$ has a local maxima.



Problem 8. If a function has a critical value at $x = 6$, does that mean that the function must have a local maxima or a local minima at $x = 6$?

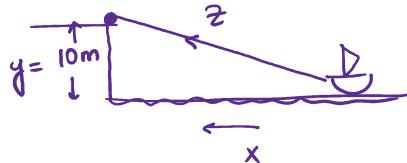
NO!



$$\frac{dz}{dt} = -30$$

Problem 9. A boat is being pulled into a dock at a rate of 30 meters per minute by a winch that is located 10 meters above the water.

- (1) How fast is the distance between the boat and the dock changing when the boat is 15 meters away from the dock?



$$\boxed{z^2 = x^2 + y^2}$$

$$\cancel{z} \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\sqrt{(15)^2 + (10)^2} \cdot -30 = 15 \cdot ? + 10 \cdot 0$$

$\because y$ is constant

$$z \frac{dz}{dt} = x \frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{z \frac{dz}{dt}}{x} = \frac{\sqrt{325} (-30)}{15}$$

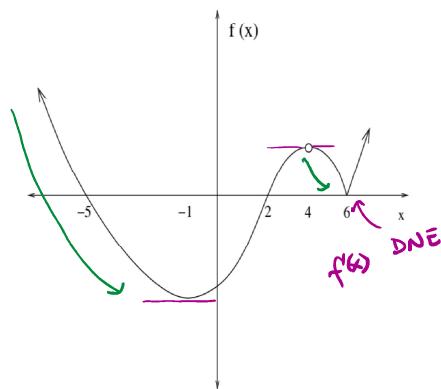
$$\frac{dx}{dt} = -36.056 \text{ meters/minute}$$

- (2) At what rate is the boat approaching the dock when the boat is 15 meters away from the dock?

$$\left| \frac{dx}{dt} \right|$$

Boat is approaching the dock at a rate of
36.056 meters per minute.

Problem 10. Answer the following questions based on the given graph:



Graph of $f(x)$ with a hole
at $x = 4$

$$D: (-\infty, 4) \cup (4, \infty)$$

- (1) Where is $f(x) < 0$? \rightarrow below the x -axis

$$(-5, 2)$$

- (2) Find the partition numbers of f' . $\rightarrow f'(x)=0$ or $f'(x)$ DNE

- (3) What are the critical points of f ?

$$x = -1, x = 6$$

$x=4$ not in D

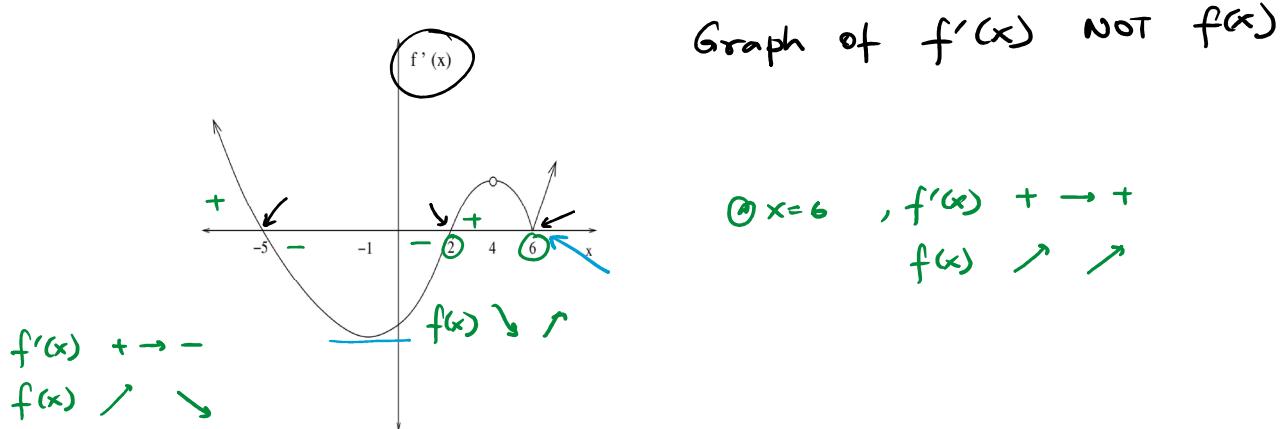
- (4) Where is $f'(x) < 0$? \rightarrow where $f(x)$ is decreasing

$$(-\infty, -1) \cup (4, 6)$$

- (5) Where are the local extrema of f ? Classify each extrema.

local minima at $x = -1$ and at $x = 6$

Problem 11. Answer the following questions based on the given graph:



(1) Find the partition numbers of f' . $\rightarrow f'(x) = 0$
 $x = -5, x = 2, x = 6$

$f'(x) \text{ DNE}$
 $x = 4$

(2) What are the critical points of f ?

$x = -5, x = 2, x = 6$

(3) Where is $f(x)$ decreasing? $\rightarrow f'(x) < 0 \rightarrow$ below x -axis
 $(-5, 2)$

(4) Where is $f'(x) \geq 0$? $(-\infty, -5) \cup (2, 4) \cup (4, 6) \cup (6, \infty)$

(5) Where are the local extrema of f ? Classify each extrema.

$x = -5 \rightarrow \text{local maxima.} \quad | \quad x = 2 \rightarrow \text{local minima.} \quad | \quad x = 6 \text{ is neither}$

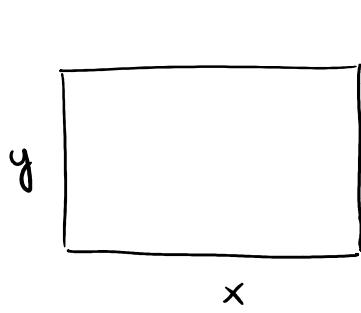
(6) Where are the local extrema of f' ? Classify each extrema.

local minima for $f'(x)$ $\circledcirc x = -1, x = 6$

but NOT $x = 4 \rightarrow$ hole .

Problem 12. The length of a 12 foot by 8 foot rectangle is increasing at a rate of 3 feet per second while its width is decreasing at a rate of 2 feet per second.

- (1) How fast is the perimeter of the rectangle changing?



Given:

$$\frac{dx}{dt} = +3 \text{ ft/s}$$

$$\frac{dy}{dt} = -2 \text{ ft/s}$$

Find : $\frac{dP}{dt}$ | $x=12$
 $y=8$

$$P = 2x + 2y$$

$$\begin{aligned}\frac{dP}{dt} &= 2 \frac{dx}{dt} + 2 \frac{dy}{dt} \\ &= (2)(3) + 2(-2) = 6 - 4 \\ &= 2 \text{ ft/sec}\end{aligned}$$

P is increasing
at a rate of
2 ft/sec

- (2) What is the rate of change of the area of the rectangle?

$$A = xy$$

$$\frac{dA}{dt} = \left(\frac{dx}{dt}\right)y + x\left(\frac{dy}{dt}\right) \rightarrow \text{product rule.}$$

$$\begin{aligned}&= (3)(8) + (12)(-2) \\ &= 24 - 24 \\ &= 0 \text{ ft/s}\end{aligned}$$

Problem 13. A company that makes deluxe toasters has a weekly demand equation given by $p(x) = 150e^{-0.02x}$, where $p(x)$ is the price per toaster in dollars, when x toasters are demanded.

- (1) What is the marginal revenue for the company when 60 toasters are sold? Interpret your answer.

- (2) At what value of x is the company's revenue increasing?