TEXAS A&M UNIVERSITY
Mathematics

Math 140 - Spring 2024
Week In Review ♯5 - Feb. 26, 2024

Section 3.4: Simplex Method

- Linear programming problem - standard form, introducing slack variables, constructing simplex tableau.
- pivot column, pivot row, and pivot element for a given simplex tableau.
- basic and non-basic variables, optimal solution, leftovers.
- Use technology to perform pivots on a simplex tableau to put the tableau in final form.

inequalities of the form
$c_1x_1 + \cdots + c_kx_k \le V$ and $V \ge 0$
$x_i \ge 0$, $x_2 \ge 0, \ldots$

Pr 1. Determine if the following linear programming problems are standard maximization problems. If they are, then convert the constraints of the linear programming problem to linear equations with slack variables, and right down the corresponding tableau.

(a)

not standard.

$$\begin{aligned} \text{maximize } & P = 2x + y \\ \text{subject to: } & 2y \le 9 - x \\ & 8 - y \le x \\ & x \ge 0, y \ge 0 \end{aligned}$$

$2y \le 9 - x$
$+x \quad +x$
$x + 2y \le 9$ ✓
$8 - y \le x$
$-x \quad -x$
$8 - y - x \le 0$
$-8 \quad -8$
$-y - x \le -8$

(b)

$$\begin{aligned} \text{Maximize } & R = y - x \\ \text{subject to: } & 3y \le 18 - 2x \\ & y - 2x + 10 \ge 0 \\ & x \ge 0, y \ge 0 \end{aligned}$$

is standard

if $a \ge 6$
then
$-a \le -6$

$3y \le 18 - 2x$
$+2x \quad +2x$
$2x + 3y \le 18$ ✓
$y - 2x + 10 \ge 0$
$-10 \quad -10$
$(-1)\big(y - 2x \ge -10 \to -10 \le y - 2x$
$-y + 2x \le 10$ ✓

Maximize $R = y - x$
$2x + 3y \le 18$
$2x - y \le 10$
$x \ge 0 \quad y \ge 0$

slack variables:
add one new variable for each non-trivial inequality
two slack variables
s_1, s_2

$2x + 3y + s_1 = 18$
$2x - y + s_2 = 10$

$0 = y - x - R$
$y - x - R = 0$
$R = y - x$
$R - y + x = 0$
$-y + x + R = 0$

x	y	s_1	s_2	R	constant
2	3	1	0	0	18
2	-1	0	1	0	10
1	-1	0	0	1	0

✓

Pr 2. For the following simplex tableau, identify the pivot row, pivot column, and pivot element.

(a)

x	y	s_1	s_2	P	constant
0	2	1	0	0	8
1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	5
0	$-\frac{5}{2}$	0	$\frac{3}{2}$	1	15

2
1/2

look @ bottom row pivot column
find the most negative entry

pivot column is 2nd col
pivot row is 1st row
compute constant / corr. column entries → denominators are positive
find smallest ratio

$\frac{8}{2} = 4$
$\frac{5}{1/2} = 5 \cdot \frac{2}{1} = 10$

$4 < 10$

pivot entry is 1st row 2nd column, value 2.

(b)

x	y	s_1	s_2	P	constant
1	0	1	0	0	8
−1	1	0	1	0	0
−2	−3	0	0	1	0

0
1

8/0 DNE
0/1 = 0

pivot column -3 is smallest
The pivot column is the 2nd column
Pivot row is the 2nd row
pivot entry is 1 in 2nd row, 2nd column.

(c)

x	y	z	s_1	s_2	P	constant
0	2	1	0	0	0	8
1	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	5
0	$\frac{1}{2}$	0	2	$\frac{3}{2}$	1	15

pivot column
no negative entries

There is no pivot.

The optimum solution has been reached.

Pr 3. For the following simplex tableau, identify the basic and non-basic variables. State the solution corresponding to the tableau, and determine if it is an optimal solution.

(a)

$$\begin{bmatrix} x & y & s_1 & s_2 & P & \text{constant} \\ 0 & 2 & 1 & 0 & 0 & 8 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 5 \\ \hline 0 & \frac{-1}{2} & 0 & \frac{3}{2} & 1 & 15 \end{bmatrix}$$

basic variables correspond to

$$\begin{matrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{matrix}, \quad \begin{matrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{matrix}, \quad \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{matrix} \quad \cdots \quad \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix}$$

delete the non-basic columns

basic variables : x, s_1, P

everything else : non-basic : y, s_2

corresponding sol'n : set non-basic variables equal to 0

not optimal there is a pivot column

$y = s_2 = 0$

x	s_1	P	const
0	1	0	8
1	0	0	5
0	0	1	15

$s_1 = 8$

$x = 5$ $\quad y = 0$

$P = 15$

$(5, 0)$ with $P = 15$

left overs $s_1 = 8$, $s_2 = 0$

(b)

$$\begin{bmatrix} x & y & s_1 & s_2 & P & \text{constant} \\ 1 & 0 & 1 & 0 & 0 & 8 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ \hline -2 & -3 & 0 & 0 & 1 & 0 \end{bmatrix}$$

ignore

not optimal

initial tableau.

basic variables : s_1, s_2, P

non-basic variables : x, y

"solution" : $x = 0$, $y = 0$ $\quad (0,0)$

with $P = 0$

(c)

$$\begin{bmatrix} x & y & z & s_1 & s_2 & P & \text{constant} \\ 0 & 2 & 1 & 0 & 0 & 0 & 9 \\ 1 & \frac{1}{2} & 0 & \frac{1}{3} & 0 & 0 & 2 \\ \hline 0 & \frac{1}{2} & 0 & 2 & \frac{3}{2} & 1 & 42 \end{bmatrix}$$

is optimal.

x	z	P	
0	1	0	9
1	0	0	2
0	0	1	42

basic variables: x, z, P

non-basic variables: s_1, s_2, y

$x = 2$ $\quad y = 0$

$z = 9$

$P = 42$

optimal solution is at $(2, 0, 9)$ with $P = 42$.

leftovers : $s_1 = s_2 = 0$, no left overs.

Pr 4. Solve.

An independent taffy company makes three flavors of taffy: strawberry, lemon, and orange. Each strawberry taffy requires 4 minutes to cool and 1 minute to wrap in paper. Each orange taffy requires 3 minutes to cool and 1.5 minutes to wrap in paper. Each lemon taffy requires 4 minutes to cool and 2 minutes to wrap in paper. There are a total of 1.5 hours available for cooling and 0.5 hours available for wrapping. Determine the production of each taffy to maximize profit if the profit on the sale of each orange, lemon, and strawberry taffy is 75 cents, 60 cents, and 50 cents, respectively, ~~and previous sales indicate that they should produce at least three times as many strawberry taffy as lemon taffy~~. How many of each flavor should the company make to maximize their profits? What is the maximum profit and is any time leftover in cooling or wrapping?

Step 1: write the linear programming problem

from last time

Step 2: Find the initial tableau

Step 3: Pivot until you can't

Step 4: Find the solution

s = number of strawberry taffy

l = no. of lemon taffies

r = no. of orange taffies

P = profit

Maximize $P = .50s + .60l + .75r$

$4s + 4l + 3r \le 90$ *

$s + 2l + 1.5r \le 30$ *

$s \ge 0$, $l \ge 0$, $r \ge 0$

$P - .50s - .60l - .75r = 0$

	s	l	r	s_1	s_2	P	const	
	4	4	3	1	0	0	90	$90/3 = 30$
pivot row →	1	2	1.5	0	1	0	30	$30/1.5 = 20$
	-.5	-.6	-.75	0	0	1	0	$30/_{3/2} = 30 \times \frac{2}{3} = 20$

pivot column

after 1 pivot

s	l	r	s_1	s_2	P	Cons.
2	0	0	1	-2	0	30
2/3	4/3	1	0	2/3	0	20
0	2/5	0	0	1/2	1	15

optimal solution

basic var's : r, s_1, P

non-basic : s, l, s_2

solution: $s = 0$, $l = 0$, $r = 20$

profit of 15.

optimum solution $(0, 0, 20)$ with profit of \$15.

leftovers $s_1 = 30$ minutes of cooling.

$s_2 = 0$ minutes of wrapping

Mathematics — Texas A&M University

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Section 4.1: Mathematical Experiments

- Sample space, **S** - a list of all possible outcomes in the mathematical experiments
- Event - a subset of the sample space
  - Simple Event
  - Certain Event
  - Impossible Event
- Using tree diagrams to determine a sample space in a two-stage experiment
- Venn Diagrams
- Operations on Events
  - Complement, A^C
  - Intersection, $A \cap B$
  - Union, $A \cup B$
- Mutually Exclusive Events

Pr 1. State the sample space for each experiment:

(a) Selecting a letter at random from the word "math" and noting the letter.

$S = \{m, a, t, h\}$

(b) Identical ping pong balls are numbered 0 to 10, one ping pong ball is drawn at random, noting the number on the ball.

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(c) A standard 20-sided die is rolled and it is noted whether the number is a multiple of 3 or is not a multiple of 3. → doesn't matter

$S = \{$multiple of 3, not a multiple of 3$\}$

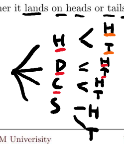
(d) The numbers 0, 1, 2, 3, and 4 are written on separate pieces of paper and put in a hat. Two pieces of paper are drawn at the same time and the product of the numbers is noted.

$S = \{0, 2, 3, 4, 2\cdot3=6, 2\cdot4=8, 3\cdot4=12\}$

$S = \{0, 2, 3, 4, 6, 8, 12\}$

(e) A card is drawn from a standard deck of 52-cards, noting the suit, and then a fair coin is flipped, noting whether it lands on heads or tails.

four suits $= \{S, C, D, H\}$



$S = \{(H,H), (H,T), (D,H), (D,T), (C,H), (C,T), (S,H), (S,T)\}$

Pr 2. Consider the experiment of selecting a letter at random from the word "math" and noting the letter.

(a) State all the simple events for the experiment. → has only one outcome in it.

simple $\{m\}$, $\{a\}$, $\{t\}$, $\{h\}$

4 simple events.

(b) State the certain event for the experiment.

all of S $\{m, a, t, h\}$

(c) Give an example of an impossible event for the experiment.

$\varnothing$ impossible event : "you pick an x".

(d) State the total number of possible events. $= 2^{\#\text{ of outcomes}}$

$2^4 = 2^{2\cdot2} = (2^2)^2 = 4^2 = 16$

(e) Write the outcomes in the event, J, "a consonant is drawn."

$J = \{m, t, h\}$

Pr 3. A card is drawn from a standard deck of 52-cards, noting the suit, and then a fair coin is flipped, noting the coin lands on heads or tails.

(a) State all the simple events for the experiment.

Caution: parentheses are required.

$\{(H,H)\}$, $\{(H,T)\}$, $\{(D,H)\}$, $\{(D,T)\}$, ... $\{(S,T)\}$

$\{S, T\}$ is not an ordered pair

(b) State the certain event for the experiment.

$S =$ whole set from 1e

$\{(H,H), (H,T), (D,H), \ldots, (S,T)\}$

(S,T) is one outcome.

(c) Give an example of an impossible event for the experiment.

- Draw a Joker, and then get Tails.

(d) State the total number of possible events.

$2^{\#\text{outcomes}} = 2^8 = 256$ possible events

(e) Write the outcomes in the event, M, "a diamond is drawn or the coin lands on heads."

$M = \{(D,H), (D,T), (H,H), (C,H), (S,H)\}$

don't include (D,H) twice.

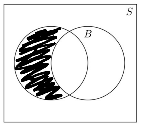
Pr 4. Let A and B be two events of the sample space, S.

Use a two-circle Venn diagram to illustrate which region(s) contain the outcomes of the resulting events.

$\cap$ = "and" $\cup$ = "or"

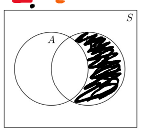
A^c = "not in A"

a. $B^C \cap A$



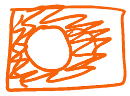
B^c

b. $(A \cup B) \cap A^C$



$A \cup B$

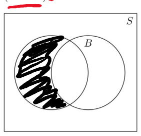
A^c



$(A^c \cup B)^c = (A^c)^c \cap B^c$

$= A \cap B^c$

c. $(A^C \cup B)^C$



$A^c \cup B$

Pr 5. An experiment consists of rolling a four-sided die, noting the number showing uppermost and then spinning a spinner with four equal regions (red, white, blue, and maroon), noting the color.

$\cup$ = union = or

$\cap$ = intersection = and

Let

V := the event "a number greater than 3 is rolled" $\{4\}$

W := the event "an even is rolled" $\{2,4\}$

X := the event "the spinner lands on blue"

Y := the event "the spinner lands on a color other than maroon" $\{$red, white, blue$\}$

Z := the event "the spinner lands on white or maroon." $\{$white, maroon$\}$

(a) Write the symbolic notation for the event, D, that "an odd is rolled or the spinner lands on white or maroon."

? or Z

$\boxed{D = W^c \cup Z}$

(b) Write the symbolic notation for the event, H, that "a number less than or equal to 3 is rolled or the spinner lands on a color other than maroon, but not blue."

≤ 3 is the opp. of > 3

Y (X^c) $V^c \cup$?

lands on blue

but = $\cup$ or $\cap$

replace "but" with and

$V^c \cup (Y \cap X^c)$

(c) Describe the event $X^C \cap W$.

not X and W

"the spinner does not land on blue and an even is rolled."

(d) Describe the event $Z \cup Y \cup Y^C$

Z or Y or Y^c

"The spinner lands on white or maroon or lands on a color other than maroon or lands on maroon."

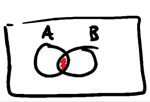
"The spinner lands on a color"

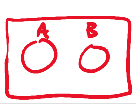
(e) Are event V and event W mutually exclusive? Explain why or why not.

mutually exclusive $V \cap W = \emptyset$

$V \cap W$ = "a 4 is rolled"

<u>Not</u> mutually exclusive.





if it is known that A, B are mutually exclusive.