Math 150 - Week-In-Review 5

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Problem Statements

1. For the function $f(x) = 2x^3 + 5$ compute and simplify the difference quotient.

$$f(x+h) = 2(x+h)^3 + 5 = 2\left[x^3 + h^3 + 3x^2h + 3h^2x\right] + 5 = 2x^3 + 2h^3 + 6x^2h + 6h^2x + 5$$

$$f(x+h) - f(x) = \cancel{2x^3} + 2h^3 + 6x^2h + 6h^2x \cancel{+5} - (\cancel{2x^3} \cancel{+5}) = 2h^3 + 6x^2h + 6h^2x$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2h^3 + 6x^2h + 6h^2x}{h} = 2h^2 + 6x^2 + 6hx$$

2. For the function $g(x) = \sqrt{x-6}$ compute and simplify $\dfrac{g(a+\Delta x) - g(a)}{\Delta x}$

$$g(a) = \sqrt{a-6}$$

$$g(a+\Delta x) = \sqrt{a + \Delta x - 6}$$

$$g(a+\Delta x) - g(a) = \sqrt{a+\Delta x - 6} - \sqrt{a-6}$$

$$\frac{g(a+\Delta x) - g(a)}{\Delta x} = \frac{\sqrt{a+\Delta x-6} - \sqrt{a-6}}{\Delta x} \cdot \frac{\sqrt{a+\Delta x-6} + \sqrt{a-6}}{\sqrt{a+\Delta x-6} + \sqrt{a-6}}$$

$$= \frac{(\cancel{a} + \Delta x - \cancel{6}) - (\cancel{a} - \cancel{6})}{\Delta x \left(\sqrt{a+\Delta x - 6} + \sqrt{a-6}\right)} = \frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{a+\Delta x-6} + \sqrt{a-6}\right)}$$

$$= \frac{1}{\sqrt{a+\Delta x - 6} + \sqrt{a-6}}$$

3. position of a particle is given by $h(t) = \dfrac{5t}{t+4}$ feet after t seconds. Find the average velocity on the interval $[\underbrace{t}_{a}, \underbrace{t+\Delta t}_{b}]$.

i.e. difference quotient

$$\text{Ave. Veloc.} = \frac{h(b) - h(a)}{b-a} = \frac{h(t+\Delta t) - h(t)}{(t+\Delta t) - t} = \frac{h(t+\Delta t) - h(t)}{\Delta t}$$

$$h(t+\Delta t) = \frac{5t + 5\Delta t}{t + \Delta t + 4}$$

$$h(t+\Delta t) - h(t) = \frac{5t + 5\Delta t}{t + \Delta t + 4} - \frac{5t}{t+4} \underset{\text{denom.}}{\overset{\text{Common}}{=}} \frac{(5t + 5\Delta t)(t+4) - (5t)(t+\Delta t+4)}{(t+\Delta t+4)(t+4)}$$

$$= \frac{\cancel{5t^2} + \cancel{20t} + \cancel{5t\Delta t} + 20\Delta t - [\cancel{5t^2} + \cancel{5t\Delta t} + \cancel{20t}]}{(t+\Delta t+4)(t+4)}$$

$$\frac{h(t+\Delta t) - h(t)}{\Delta t} = \frac{\frac{20\Delta t}{(t+\Delta t+4)(t+4)}}{\Delta t} = \frac{20}{(t+\Delta t+4)(t+4)}$$

4. Solve for h in the following equation.

$$\left|\frac{1}{h+3}+2\right| = \left|\frac{2}{(h-1)(h+3)}\right|$$

$\rightsquigarrow h \neq 1$ & $h \neq -3$

① $\dfrac{1}{h+3} + 2 = \dfrac{2}{(h-1)(h+3)}$

$(h-1)(h+3) \times \left(\dfrac{1}{h+3} + 2\right) = \left(\dfrac{2}{(h-1)(h+3)}\right) \times (h-1)(h+3)$

$h-1 + 2(h-1)(h+3) = 2$

$h-1 + 2\left[h^2 + 2h - 3\right] = 2$

$h-1 + 2h^2 + 4h - 6 = 2$

$2h^2 + 5h - 7 = 2$

$2h^2 + 5h - 9 = 0$

$$\boxed{h = \frac{-5 \pm \sqrt{97}}{4}}$$

② $\dfrac{1}{h+3} + 2 = \dfrac{-2}{(h-1)(h+3)}$

$(h-1)(h+3) \times \left(\dfrac{1}{h+3} + 2\right) = \left(\dfrac{-2}{(h-1)(h+3)}\right) \times (h-1)(h+3)$

$h-1 + 2(h-1)(h+3) = -2$

$h-1 + 2\left[h^2 + 2h - 3\right] = -2$

$h-1 + 2h^2 + 4h - 6 = -2$

$2h^2 + 5h - 7 = -2$

$2h^2 + 5h - 5 = 0$

$$\boxed{h = \frac{-5 \pm \sqrt{65}}{4}}$$

5. Solve for v in the following equation.

$$\frac{v+4}{v+1} - \frac{v+5}{v-1} = -1$$

$v \neq -1$, $v \neq 1$

$\times(v+1)(v-1)$

$(v+4)(v-1) - (v+5)(v+1) = -(v-1)(v+1)$

$v^2 + 3v - 4 - (v^2 + 6v + 5) = -v^2 + 1$

$-3v - 4 - 5 = -v^2 + 1$

$v^2 - 3v - 10 = 0$

$(v-5)(v+2) = 0$

$v = 5$

& $v = -2$

6. For the following function, state the domain, identify the intercepts, analyze the end behavior and sketch the graph.

$$f(x) = \sqrt{(1+3x)(1-x^2)}$$

Domain $(1-3x)(1-x^2) \geq 0 \Rightarrow x = 1/3 \;\&\; 1 \;\&\; -1$

$+ \quad - \quad + \quad -$

$-1 \quad -\frac{1}{3} \quad 1$

$x \in (-\infty, -1] \cup [-\frac{1}{3}, 1]$

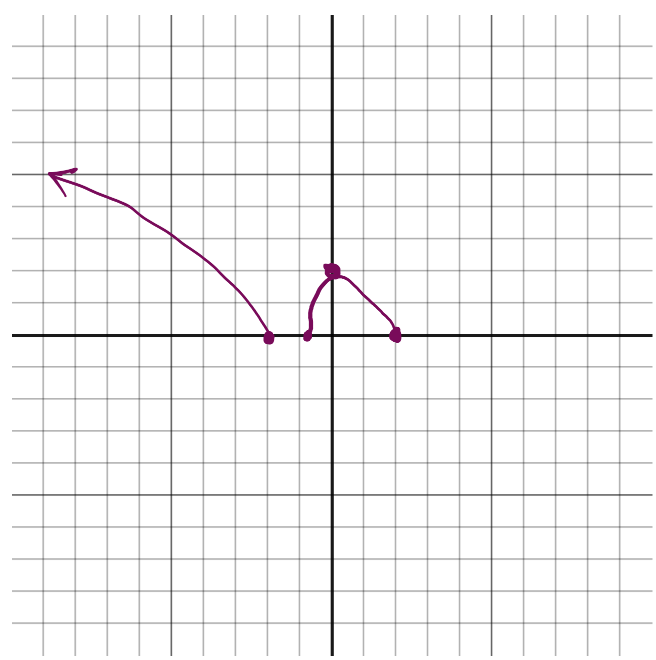
x-int. let $y=0 \rightarrow x = -\frac{1}{3}, 1, -1$ $\quad (1,0) \quad (-1,0) \quad (-\frac{1}{3}, 0)$

Y-int. let $x=0 \rightarrow \sqrt{(1+0)(1-0)} = \sqrt{1} = 1$ $\quad (0,1)$

End behavior

as $x \rightarrow -\infty \quad f(x) \rightarrow \infty$ (no Horiz. Asy.)

Note: $\sqrt{\#} \geq 0$ so always above x-axis. (no reflections w.r.t x-axis)



7. For the following function, state the domain, identify the intercepts, analyze the end behavior and sketch the graph.

$$g(x) = \frac{2x}{(1+3x)^{\frac{1}{5}}}$$

Domain: $1+3x \neq 0 \quad x \neq -\frac{1}{3} \Rightarrow x \in (-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, \infty)$ Vertical Asy. at $x = -\frac{1}{3}$

x-int. $\rightarrow$ let $y=0$ $\quad \frac{2x}{\sqrt[5]{(1+3x)}} = 0 \rightarrow 2x = 0 \rightarrow (0,0)$

y-int. $\rightarrow$ let $x=0$ $\quad \frac{2(0)}{\sqrt{1+3(0)}} = 0 \rightarrow (0,0)$

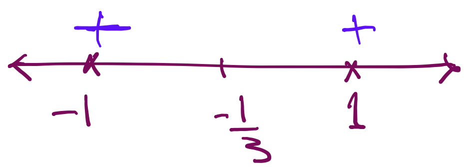
End behavior:

$$\frac{2x}{\sqrt[5]{1+3x}} \underset{\approx}{\text{leading term}} \frac{2x}{\sqrt[5]{3x}} = \frac{2x}{(3x)^{\frac{1}{5}}} = \frac{2x^{1-\frac{1}{5}}}{(3)^{\frac{1}{5}}} = \frac{2}{\sqrt[5]{3}} \cdot x^{\frac{4}{5}} \Rightarrow \text{No H.A.}$$

or

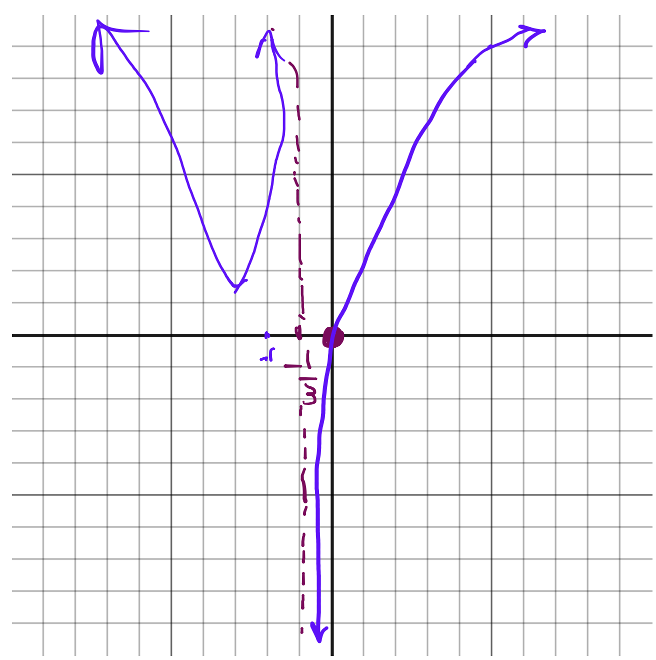
$$\frac{2x}{\sqrt[5]{1+3x}} \cdot \frac{\frac{1}{\sqrt[5]{3x}}}{\frac{1}{\sqrt[5]{3x}}} = \frac{\frac{2x}{\sqrt[5]{3x}}}{\sqrt[5]{\frac{1}{3x} + \frac{3x}{3x}}} = \frac{\frac{2x}{\sqrt[5]{3x}}}{\sqrt[5]{\frac{1}{3x}+1}} \rightarrow \frac{\frac{2x}{\sqrt[5]{3x}}}{1}$$

$$= \frac{2x}{\sqrt[5]{3x}} = \frac{5x^{4/5}}{\sqrt[5]{3}}$$



$$g(-1) = \frac{-2}{\sqrt[5]{1-3}} = \frac{-2}{(-2)^{\frac{1}{5}}} = \frac{-}{-} = +$$

$$g(1) = \frac{2}{\sqrt[5]{1+3}} = +$$



8. For the following function, state the domain, identify the intercepts, analyze the end behavior and sketch the graph.

$$h(x) = \sqrt{x+3}\,(1+3x)^{\frac{1}{5}}$$

No restriction

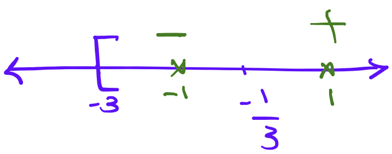
Domain: $x \geq -3$ $\quad x \in [-3, \infty)$

x-int.: $x = 0$ $\quad \sqrt{3}\,(1+0)^{\frac{1}{5}} = \sqrt{3}$ $\quad (0, \sqrt{3})$

Y-int.: $y = 0 \rightarrow x = -3$ & $x = -\frac{1}{3}$ $\quad (-\frac{1}{3}, 0)$ $\quad (-3, 0)$

End behavior

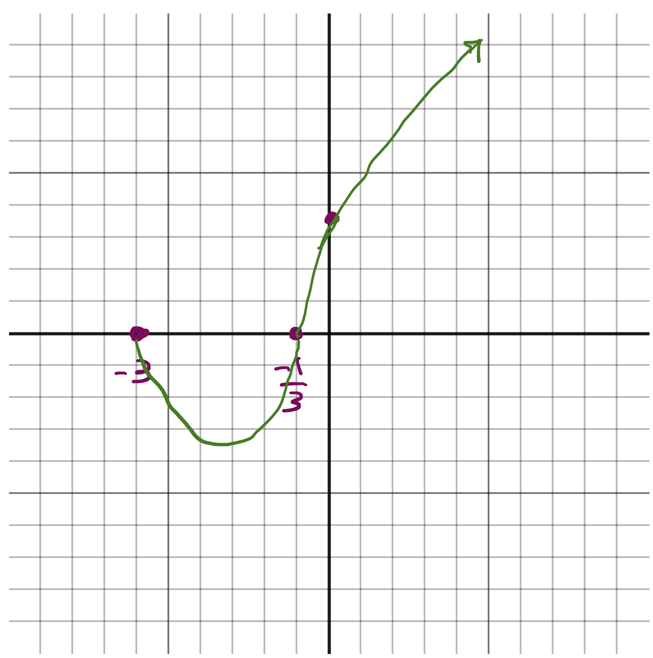
as $x \rightarrow \infty$, $f(x) \rightarrow \infty$



$h(-1) = \sqrt{2}\,(1-3)^{\frac{1}{5}} = \underbrace{\sqrt{2}}_{+}\,\underbrace{(-2)^{\frac{1}{5}}}_{-} < 0$

$h(1) = \sqrt{4}\,(1+3)^{\frac{1}{5}} > 0$

unusual steepness at $x = -\frac{1}{3}$



9- Write P as a function of t

i.e. solve for P

$$t = \frac{2+t^2}{\sqrt{3P-8}}$$

we should free the P

$$t\left(\sqrt{3P-8}\right) = 2+t^2$$

$$t^2(3P-8) = (2+t^2)^2$$

$$t^2(3P-8) = 4+t^4+4t^2$$

$$3P-8 = \frac{4+t^4+4t^2}{t^2}$$

$$3P-8 = \frac{4}{t^2}+t^2+4$$

$$3P = \frac{4}{t^2}+t^2+12$$

$$\boxed{P = \frac{4}{3t^2}+\frac{t^2}{3}+4}$$