Math 308: Week-in-Review 5

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Review for Exam 1

1. For the initial value problem $(t^2 - 4)y' + 2ty = 3t^2, \qquad y(1) = -3$

   (a) Determine an interval in which the solution to the initial value problem is certain to exist.

   (b) Solve the initial value problem.

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2. A tank initially contains 10 L of fresh water. Brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution is kept well stirred and flows out of the tank at a rate of 2 L/min. Determine the concentration of salt in the tank as a function of time.

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3. Given the differential equation $\dfrac{dy}{dt} = 7y - y^2 - 10$

   (a) Find the equilibrium solutions.

   (b) Sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable.

   (c) Sketch the graph of some solutions.

   (d) Determine the behavior of $y(t)$ as t increases for all possible values of $y(0) = y_0$.

   (e) Solve the equation.

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4. Solve the following equations. If any initial value is given, then solve the initial value problem. If no initial value is given, then find the general solution. Find an explicit solution if possible.

   (a) $y' - 2y = x^2 e^{2x}$

   (b) $f'' - 7f' + 12f = 2e^{5t}, \quad f(1) = 0, \;\; f'(1) = -1$

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(c) $(4t - 2y)y' = 4t - 4y$

(d) $y' = ty^2 - t$

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(e) $g'' + 2g' + 2g = 2t, \quad g(0) = 0, \;\; g'(0) = 1$

(f) $u'' + 2u' + u = 2e^{-t}$

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5. Solve the differential equation by finding an integrating factor that makes the equation exact

$$e^x y' = e^{3x} + e^x y - e^x$$

6. Verify that $y_1(t) = t$ is a solution of the differential equation

$$t^2 y'' - t(t+2)y' + (t+2)y = 2t^3, \quad (t > 0)$$

then find a second solution y_2 so that $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions.

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7. Without solving the initial value problem, determine an interval in which the solution is guaranteed to exist.

$$y' + \ln(t+3)y = \sqrt{16 - t^2}, \quad y(-1) = 3.$$

8. Without solving the initial value problem, state for which values of t_0 and y_0 the initial value problem is guaranteed to have a unique solution in at least some small interval around t_0.

$$y' = \sqrt{1 - t^2 - y^2}, \quad y(t_0) = y_0$$

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9. Solve the initial value problem and determine the interval where the solution exists

$$y' = 3ty^2, \;\; y(0) = y_0$$

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