Note ♯4: Exam 02 Review

Problem 1. Find and sketch the domain of the function $f(x,y) = \frac{1}{\sqrt{9-x^2-y^2}} + \sqrt{x+y}$.

Problem 2. Find the directional derivative of $f(x,y,z) = xye^z$ at the point $P(2,4,0)$ in the direction of $Q(3,2,1)$.

Problem 3. Consider the function $f(x,y,z) = x^2 - 2y^2 + 3z^2 + xy$ and a point $P(2,-2,1)$. Find the maximum rate of change of f at the point P and the direction in which it occurs.

Problem 4. Find an equation of the tangent plane to the given surface at the specified point.

$$z = x\sin(x+y), \quad (-1,1,0)$$

Problem 5. Use the linear approximation(or differentials) of the function $f(x,y) = 1 - xy\cos(\pi y)$ to approximate $f(1.03, 0.98)$.

Problem 6. Describe the level curves for

(1) $f(x,y) = 3x - y$
(2) $f(x,y) = \sqrt{x^2+2y^2}$

Problem 7. The base radius and height of a circular cylinder are measured as $2cm$ and $5cm$, respectively, with possible errors in measurements of as much as $0.1cm$ and $0.2cm$, respectively. Use the differentials to estimate the maximum error in the calculated volume of the cylinder.

Problem 8. Find f_{yx} of $f(x,y) = \cos(x^2y)$.

Problem 9. Use the chain rule to calculate $\frac{\partial z}{\partial s}$ when $(s,t) = (1,2)$.

$$z = xe^{xy}, \quad x = st - 1, \quad y = s^2 + t$$

Problem 10. The length x, width y and height z of a box change with time. At a certain instant, the dimensions are $x = 2m$, $y = 3m$, and $z = 4m$, and x, y are increasing at rates $5m/s$, $2m/s$ respectively, while z is decreasing at a rate of $1m/s$. At that same instant, find the rate at which the length of a diagonal is changing.

Problem 11. Consider the surface

$$2(x-1)^2 + 3(y-3)^2 - (z-2)^2 = 2, \quad z \geq 0$$

at the point P on the surface determined by $(x,y) = (3,2)$.

(1) Find the equation of the tangent plane in the form of a linear equation ($ax+by+cz+d=0$).
(2) Find the equation of the normal line in the form of parametric equations.

Problem 12. Find all critical points and classify them as local maximum, local minimum, or saddle point. You don't have to find the values of f.

$$f(x,y) = \frac{2}{3}x^3 + xy^2 + 3x^2 + 2y^2$$

Problem 13. Let $f(x,y) = x^2 + 2y^2 - 2x - 4y + 1$. Find the absolute maximum and minimum over the triangle with vertices $(0,0), (2,0)$, and $(2,4)$.

Problem 14. Find the absolute maximum and minimum values of f on the region described by the inequality. Use the Lagrange multiplier method.

$$f(x,y) = 2x^2 + 3y^2, \quad 3x^2 + y^2 \leq 12$$