Note ♯6: Sections 15.6-15.8

Problem 1. Let E be the region bounded by $y = x^2$ and $x = y^2$ and $z = 0$ and $z = 5x + 5y$. Compute $\iiint_E 4xy\, dV$. Just set up without evaluation.

Problem 2. Evaluate the triple integral $\iiint_T y^2 dV$, where T is the solid tetrahedron with vertices $(0,0,0), (2,0,0), (0,2,0)$, and $(0,0,2)$.

Problem 3. Evaluate $\iiint_E \sqrt{y^2+z^2}\, dV$, where E is the solid between the elliptic paraboloids $x = y^2 + z^2$ and $x = 18 - y^2 - z^2$.

Problem 4. Express $\iiint_E f(x,y,z)dV$ in the order $dydzdx$ if E is the solid bounded by $y = x^2$, $z = 0$, $y + 4z = 16$.

Problem 5. Rewrite the integral as an equivalent iterated integral in the five other orders.

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)dzdydx$$

Problem 6. (a) Plot the point whose cylindrical coordinates are $(\sqrt{2}, 3\pi/4, 2)$. Then find the rectangular coordinates of the point.

(b) Change from rectangular to cylindrical coordinates of the point $(-2, 2\sqrt{3}, 3)$.

Problem 7. Convert $\int_{-2}^0 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 xz\, dzdydx$ to cylindrical coordinates.

Problem 8. Find the volume of the sold that is above the xy plane, below the ellipsoid $4x^2+4y^2+z^2 = 64$ but inside the cylinder $x^2 + y^2 = 9$.

Problem 9. Find the volume of the solid that is enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 12$ and $z = 2$.

Problem 10. (a) Plot the point whose spherical coordinates are $(4, -\pi/4, \pi/3)$. Then find the rectangular coordinates of the point.

(b) Change from rectangular to spherical coordinates of the point $(-1, 1, -\sqrt{2})$.

Problem 11. Convert $\int_{-3}^3 \int_0^{\sqrt{9-y^2}} \int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}dzdxdy$ to spherical coordinates.

Problem 12. Using spherical coordinates, find the volume of the part of the ball $\rho \leq 3$ that lies between the cones $\phi = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$.

Problem 13. Evaluate $\iiint_E z^2 dV$, where E is bounded by the xy-plane and the hemispheres $z = \sqrt{1-x^2-y^2}$ and $z = \sqrt{9-x^2-y^2}$.