

Session 8

Monday, March 25, 2024 5:24 PM



Math 140 - Spring 2024
WEEK IN REVIEW #7 - MAR. 25, 2024

SECTION 5.2 PART A: POLYNOMIAL FUNCTIONS

- General Notation of a Polynomial

- Degree

- Leading Coefficient

- Constant Term

- End Behavior

- Domain \leftarrow easy

- Intercepts

- Parent Polynomial Functions

- Zero $f(x) = 0$

- Constant $f(x) = b$, where $b \neq 0$

- Linear $f(x) = x$

- Quadratic $f(x) = x^2$

- Cubic $f(x) = x^3$

$$\left. \begin{array}{l} \text{sums of constants times powers of } x \\ \text{whole numbers} \end{array} \right\} \rightarrow f(x) = x^3 - 3x + 2$$

↑ degree: 3
leading coeff: 1
constant term: 2

- Pr 1. Determine if the given function is a polynomial function. If the answer is yes, state the degree, leading coefficient, and constant term.

(a) $f(x) = -42x^{-1} + 3x^{0.5} - 6x^{3.1}$

not a polynomial.

(b) $g(w) = \sqrt{3}w^2 - w^3 + \frac{1}{7}w - 21w^0$

L is fine

$w^3 + \sqrt{3}w^2 + \frac{1}{7}w - 21$

this is a polynomial of degree 3.

leading coefficient: -1

constant term: -21

(-w^3 = +(-1)w^3)

- Pr 2. Describe the end behavior of each polynomial function, both symbolically and with a quick sketch of the end behavior.

(a) $f(x) = -x^2 - 6x - 2048$

degree is 4, is even
lead coef is -1

as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$. as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

the degree is

case i: even, lead coef pos.

as $x \rightarrow \infty$, $p(x) \rightarrow \infty$

as $x \rightarrow -\infty$, $p(x) \rightarrow \infty$

$f(x) = x^2$

as $x \rightarrow \infty$, $p(x) \rightarrow \infty$

as $x \rightarrow -\infty$, $p(x) \rightarrow \infty$

case ii: degree even, lead coef < 0

$f(x) = -x^2$

as $x \rightarrow \infty$, $p(x) \rightarrow -\infty$

as $x \rightarrow -\infty$, $p(x) \rightarrow -\infty$

case iii: degree odd, lead coef > 0

$f(x) = x$

as $x \rightarrow \infty$, $p(x) \rightarrow \infty$

as $x \rightarrow -\infty$, $p(x) \rightarrow -\infty$

case 4: degree odd, lead coef < 0

$f(x) = -x$

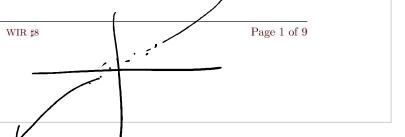
as $x \rightarrow \infty$, $p(x) \rightarrow -\infty$

as $x \rightarrow -\infty$, $p(x) \rightarrow \infty$

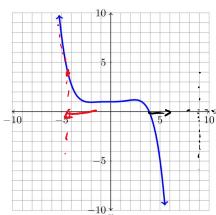
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- Pr 3. Describe the end behavior symbolically for the polynomial function, $f(x)$, graphed below.



as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$
as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$.

- Pr 4. State the domain of each polynomial function.

(a) $f(x) = 2x^{13} - 6x^2 - 40x$

$(-\infty, \infty)$ ✓

(b) $g(w) = 15w^2 - w^3 + 5w - 12$

$(-\infty, \infty)$ ✓

$$x=a, b \quad (x, 0) \quad (0, y)$$

Pr 5 Determine all exact real zeros, the x -intercept(s), and y -intercept of each given polynomial function, if possible.

$$(a) f(x) = -5(2-3x)(4x+9)$$

$$-5(2-3x)(4x+9) = 0$$

$$-5 \neq 0 \quad 2-3x=0 \quad 4x+9=0$$

$$\begin{array}{r} +3x \\ \hline 2 = 3x \end{array} \quad \begin{array}{r} -9 \\ \hline 4x = -9 \end{array}$$

$$(b) g(x) = 3x^3 - 3x^2 - 18x = 3x(x^2 - x - 6)$$

$$3x(x+3)(x-2) = 0$$

$$3x=0 \quad x+3=0 \quad x-2=0$$

$$\downarrow \quad \begin{array}{r} -3 \\ -3 \\ \hline 2x = -3 \end{array} \quad \begin{array}{r} +2 \\ +2 \\ \hline x=2 \end{array}$$

$$x=0 \quad x=-3 \quad x=2$$

$$(c) h(w) = 5w^2 - w^3 + 4w - 20 =$$

$$\begin{array}{r} -w^3 + 5w^2 + 4w - 20 \\ \hline \end{array}$$

$$(-w^3 + 5w^2) + (4w - 20) = -w^2(w-5) + 4(w-5) \quad \because w$$

"rational roots test"

$$w=2$$

$$(d) k(x) = (x^2+9)(x^2-4)$$

$$(x^2+9)(x^2-4) = 0$$

$$x^2+9=0 \quad x^2-4=0$$

$$\downarrow \quad x^2=4$$

$$x^2=-9$$

$$x = \pm \sqrt{-9}$$

$$= \pm 3\sqrt{-1}$$

$$= \pm 3i$$

$$\text{negatives} \neq \text{real}$$

$$(x, 0) \quad (0, y)$$

$$\text{if } ab=0, \text{ then } a=0 \text{ or } b=0$$

$$x = -\frac{9}{4}, \frac{2}{3}$$

$\frac{9}{4}$ is not "exact"

x -intercepts are $(\frac{2}{3}, 0), (-\frac{9}{4}, 0)$

y -intercept: $(0, f(0)) = (0, -90)$

$$f(0) = -5(2-3 \cdot 0)(4 \cdot 0 + 9)$$

$$= -5(2)(9) = -90$$

Zeros: $x = -3/2, 0, \text{ or } 2$

x -intercepts: $(-3/2, 0), (0, 0), (2, 0)$

y -intercept: $(0, 0)$

y -intercept: $(0, -20)$

$(-w^3 + 5w^2) + (4w - 20) = -w^2(w-5) + 4(w-5) \quad \because w$

$= (-w^2 + 4)(w-5) \quad w = -2, 2, 5$

$w=5$ x -intercepts

$(-2, 0), (2, 0), (5, 0)$

$-w^2 + 4 = 0$

$-4 = w^2 \rightarrow w = \pm 2$

$x = -2, 2$

x -intercepts: $(-2, 0), (2, 0)$

y -intercept: $(0, -36)$

$k(0) = (0^2+9)(0^2-4)$

$= 9 \cdot (-4)$

$= -36$

SECTION 5.2 PART B: QUADRATIC FUNCTIONS

- General form of a Quadratic Function - $f(x) = ax^2 + bx + c$ where a, b , and c are real numbers with $a \neq 0$
 - Vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
 - Axis of Symmetry $x = -\frac{b}{2a}$
 - Domain and Range $[-\infty, \infty]$
- Quadratic Formula - used to solve equations of the form $ax^2 + bx + c = 0$ - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Recall: Profit = Revenue - Cost

$$\begin{array}{c} \text{if } a > 0 \\ \text{graph: open upwards} \\ \text{vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \text{ or } \left[-\infty, f\left(-\frac{b}{2a}\right)\right] \\ \text{if } a < 0 \\ \text{graph: open downwards} \\ \text{vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \end{array}$$

- Pr 1. Determine the vertex, axis of symmetry, domain, range, x -intercept(s), y -intercept, maximum value and minimum value for each quadratic function, if they exist.

(a) $f(x) = 2x^2 + 6x$

• axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2 \cdot 2} = -\frac{6}{4} = -\frac{3}{2}$

• range: $[-\frac{9}{4}, \infty)$

• minimum value is $-\frac{9}{4}$, for the vertex, $f\left(-\frac{b}{2a}\right) = f\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right)^2 + 6\left(-\frac{3}{2}\right)$

• there is no maximum value. vertex: $\left(-\frac{3}{2}, -\frac{9}{4}\right) = 2 \cdot \frac{3^2}{2^2} - \frac{6 \cdot 3}{2}$

• y -intercept: $f(0) = c = 0$

(0,0)

• x -intercept: $f(x) = 0 = 2x^2 + 6x$

$= 2x(x+3)$

$x=0$

$x+3=0$

$x=-3$

(0,0), (-3,0)

• x -intercept: (0,0), (-3,0)

• domain: $(-\infty, \infty)$

• range: $[0, \infty)$

• minimum value is 0

• maximum value does not exist.

• x -intercepts: $g(x)=0$

$3x^2 - 6x + 3 = 0$

$= 3(x^2 - 2x + 1)$

$= 3(x-1)(x-1) = 3(x-1)^2 = 0$

$x-1=0 \rightarrow x=1$

(1,0)

• y -intercept: (0,3)

$g(0) = c = 3$.

(c) $h(x) = 36 - 49x^2 = -49x^2 + 0x + 36$

• axis of symmetry:

$x = -\frac{b}{2a} = 0$

• vertex: $h(0) = 36$

vertex: (0,36)

• domain: $(-\infty, \infty)$

• range: $(-\infty, 36]$

• maximum value is 36.

• There is no minimum value.

• $a = -49 < 0$

$b = 0$

$c = 36$

• y -intercept: (0,36)

• x -intercept: $(-\frac{6}{7}, 0), (\frac{6}{7}, 0)$

$-49x^2 + 36 = 0$

$+49x^2 + 49x^2$

$36 = 49x^2$

$\frac{49x^2}{49} = \frac{36}{49}$

$x^2 = \frac{36}{49} = \frac{6^2}{7^2}$

$x = \pm \sqrt{\frac{36}{49}} = \pm \frac{6}{7}$

$a = \frac{1}{5} > 0$

$b = \frac{49}{500}$

$c = -\frac{31}{100}$

• axis of symmetry

$x = -\frac{b}{2a} = -\frac{\frac{49}{500}}{2 \cdot \frac{1}{5}} = -\frac{\frac{49}{500}}{\frac{2}{5}} = -\frac{49}{500} \cdot \frac{5}{2} = -\frac{49}{200}$

$x = -\frac{49}{200}$

• vertex $f\left(-\frac{49}{200}\right) = \frac{1}{5} \left(\frac{-49}{200}\right)^2 + \frac{49}{500} \left(\frac{-49}{200}\right) - \frac{31}{100}$

$= \frac{(-49)^2}{200^2 \cdot 5} - \frac{49 \cdot 49}{200 \cdot 500} - \frac{31}{100}$

$= \frac{(-49)^2}{200000} - \frac{49^2 \cdot 2}{100000 \cdot 5} - \frac{31 \cdot 100}{100000} \cdot \frac{2}{2}$

$= \frac{49^2}{200000} - 2 \cdot \frac{49^2}{100000} - \frac{31 \cdot 100}{100000}$

$= \frac{49^2}{200000} - \frac{2 \cdot 49^2}{100000} - \frac{31 \cdot 100}{100000}$

$= \frac{-49^2 - 62000}{200000} = -\frac{64401}{200000}$

• domain: $(-\infty, \infty)$

range: $[-\frac{64401}{200000}, \infty)$

• minimum value is $-\frac{64401}{200000}$

• no maximum value

• y -intercept: (0, -31/100)

• x -intercept:

$\frac{1}{5}x^2 + \frac{49}{500}x - \frac{31}{100} = 0$

$x = -\frac{49}{500} \pm \sqrt{\left(\frac{-49}{500}\right)^2 - 4 \cdot \frac{1}{5} \cdot \left(\frac{-31}{100}\right)}$

will finish

$$= \frac{49^2 - 2 \cdot 49^2 - 62000}{200000}$$

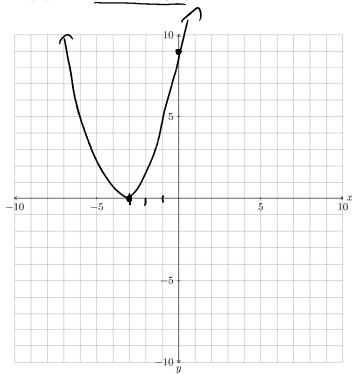
$$= \frac{-49^2 - 62000}{200000} = \frac{-64401}{200000}$$

vertex: $\left(-\frac{49}{200}, -\frac{64401}{200000} \right)$



Pr 2. Graph the quadratic function with the following properties

- i. As $x \rightarrow -\infty$, $h(x) \rightarrow \infty$ and as $x \rightarrow \infty$, $h(x) \rightarrow \infty$
- ii. $h(x)$ has a single real zero of -3 .
- iii. There is a minimum value of 0 .
- iv. The graph has a y-intercept of $(0, 9)$.



$$\frac{1}{5}x^2 + \frac{1}{500}x - \frac{31}{100} = 0$$

$$x = \frac{-49}{500} \pm \sqrt{\left(\frac{-49}{500}\right)^2 - 4 \cdot \frac{1}{5} \cdot \left(\frac{-31}{100}\right)}$$

will finish
later

$$= \frac{-49}{500} \pm \sqrt{\frac{49^2}{500^2} + \frac{124}{500}}$$

$$= \frac{-49}{500} \pm \frac{1}{500} \sqrt{49^2 + 62000}$$

$$= \frac{-49 \pm \sqrt{64401}}{200}$$

Pr 3. Use the given revenue function, $R(x)$, and cost function, $C(x)$, where x is the number of items made and sold, to determine each of the following. Assume both revenue and cost are given in dollars.

- The number of items sold when revenue is maximized.
- The maximum revenue.
- The number of items sold when profit is maximized.
- The maximum profit.
- The break-even quantity (quantities).

(a) $R(x) = -10x^2 + 370$ and $C(x) = 20x + 660$

$$i) x = \frac{-b}{2a} = \frac{0}{2(-10)} = 0 \quad a = -10, b = 0, c = 370$$

revenue is maximized when 0 items are sold.

$$ii) R(0) = -10 \cdot 0^2 + 370 = 370$$

maximum revenue is \$370

$$P(x) = R(x) - C(x) = -10x^2 + 370 - (20x + 660)$$

$$= -10x^2 - 20x + 370 - 660$$

$$= -10x^2 - 20x - 290$$

$$iii) x = \frac{-b}{2a} = \frac{-(20)}{2(-10)} = \frac{20}{20} = 1$$

profit is maximized when 1 item is sold.

$$iv) P(-1) = -10(-1)^2 - 20(-1) - 290$$

$$= -10 + 20 - 290$$

$$= -280 \quad \text{max. profit is } \$-280.$$

(b) $R(x) = -x^2 + 24x$ and $C(x) = x + 10$

$$i) x = \frac{-b}{2a} = \frac{-24}{2(-1)} = 12$$

12 items

$$ii) R(12) = -(12)^2 + 24 \cdot 12$$

$$= -12 \cdot 12 + 24 \cdot 12$$

$$= (-12+24) \cdot 12 = 12 \cdot 12 = \$144$$

maximum revenue is \$144.

$$iii) P(x) = R(x) - C(x) = -x^2 + 24x - (x + 10)$$

$$x = -\frac{b}{2a} = -\frac{23}{2(-1)} = \frac{23}{2}$$

$\frac{23}{2}$ units

cost: \$18 a bottle

Pr 4. The cost to produce bottled mineral water is given by $C(x) = 18x + 7500$, where x is the number of thousands of bottles produced. The profit from the sale of these bottles is given by the function $P(x) = -x^2 + 300x - 7500$.

- (a) How many bottles must be sold to maximize the profit?

find the x-coordinate of the vertex.

$$x = -\frac{b}{2a} = -\frac{300}{2(-1)} = \frac{300}{2} = 150$$

150 bottles

- (b) What is the maximum profit? $\rightarrow P\left(-\frac{b}{2a}\right)$

$$P(150) = -150^2 + 300 \cdot 150 - 7500$$

$$= \$15000$$

- (c) What is the revenue when the profit is maximized?

Find $R(150)$

$$R(x) = P(x) + C(x) = -x^2 + 300x - 7500 + 18x + 7500$$

$$P(x) = R(x) - C(x)$$

$$R(x) = -x^2 + 318x = x(-x + 318)$$

$$R(150) = -150^2 + 318 \cdot 150 = \$25,200$$

$$\text{alternatively, } R(150) = P(150) + C(150) \\ = 15000 + 18 \cdot 150 + 7500$$

- v) break-even quantities

$$\text{when } C(x) = R(x) \quad \text{or } P(x) = 0$$

$$-10x^2 - 20x - 290 = 0$$

no break-even point!

$$-10(x^2 + 2x + 29) = 0$$

$$x^2 + 2x + 29 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 29}}{2}$$

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 29}}{2} = \frac{-2 \pm \sqrt{\text{negative}}}{2}$$

$$vi) P\left(\frac{23}{2}\right) = -\left(\frac{23}{2}\right)^2 + 23 \cdot \frac{23}{2} - 10$$

$$= -\frac{23^2}{4} + \frac{23^2}{2} - 10$$

$$= 23^2 \left(-\frac{1}{4} + \frac{1}{2}\right) - 10$$

$$= 23^2 \cdot \frac{1}{4} - 10$$

$$= \frac{23^2 - 40}{4} = \frac{529 - 40}{4}$$

$$\text{maximum is } \frac{489}{4} = \$122.25$$

- v) break-even quantity

$$P(x) = 0$$

$$-x^2 + 23x - 10 = 0$$

$$x^2 - 23x + 10 = 0$$

$$x = \frac{(-23) \pm \sqrt{(-23)^2 - 4 \cdot 1 \cdot 10}}{2}$$

$$= \frac{23 \pm \sqrt{529 - 40}}{2}$$

$$x = \frac{23 \pm \sqrt{489}}{2} \approx \frac{45}{2} \text{ or } \frac{1}{2}$$

alternatively, $R(150) = P(150) + C(x)$

$$= 15000 + 18 \cdot (150) + 7500$$

Pr 5. The cost of manufacturing collectible bobble head figurines is given by $C(x) = 30x + 350$, where x is the number of collectible bobble head figurines produced. If each figurine has a price-demand function of $p(x) = -1.2x + 360$, in dollars, determine

(a) the company's profit function.

$$P(x) = R(x) - C(x), \quad R(x) = p(x) \cdot x$$

$$P(x) = (-1.2x + 360)x - (30x + 350)$$

$$= -1.2x^2 + 360x - 30x - 350$$

$$\boxed{P(x) = -1.2x^2 + 330x - 350}$$

(b) how many figurines must be sold in order to maximize revenue?

$$R(x) = -1.2x^2 + 360x$$

$$x = \frac{-b}{2a} = \frac{-360}{2 \cdot (-1.2)} = \frac{360}{2.4} = \frac{3600}{24} = \frac{1200}{8} = \frac{300}{2}$$

$$\boxed{x = 150 \text{ figurines}}$$

(c) how many figurines must be sold in order to maximize profit?

$$x = \frac{-b}{2a} = \frac{-330}{2(-1.2)} = \frac{330}{2.4} = 137.5$$

$$= 137.5$$

$$\boxed{x = 137.5 \text{ figurines}}$$

(d) at what price per figurine will the maximum profit be achieved?

price - demand function

$$\rightarrow P(137.5) = P\left(-\frac{b}{2a}\right) = P(137.5)$$

lower case

$$= -1.2(137.5) + 360$$

$$= \boxed{\$195}$$