Math 150 - Week-In-Review 1

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Problem Statements

1. Consider the function

$$h(x) = \begin{cases} -2x+4 & \text{, if } x \leq -1 \\ (x-2)^2 & \text{, if } x > -1. \end{cases}$$

Find $h(-2)$, $h(-1)$, and $h(2)$.

input

$h(-2) = -2(-2) + 4 = 4 + 4 = 8$

$h(-1) = -2(-1) + 4 = 6$

$h(2) = (2-2)^2 = 0$

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2. Find an equation of the line through the points $(3,9)$ and $(-7,1)$ in standard form.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(9)}{-7-3} = \frac{-8}{-10} = \frac{4}{5}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 9 = \frac{4}{5}(x-3)}$$ Point-slope form

$$y - 9 = \frac{4}{5}x - \frac{12}{5} \longrightarrow y = \frac{4}{5}x + 9 - \frac{12}{5} \dashrightarrow y = \frac{4}{5}x + \frac{\overbrace{45-12}^{33}}{5}$$

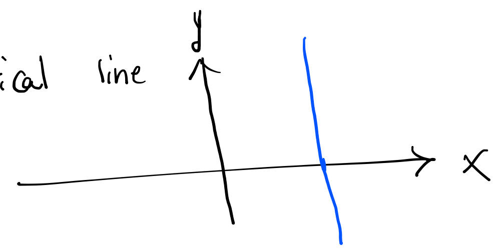
$$y = \frac{4}{5}x + \frac{33}{5}$$ slope intercept form

$$y - \frac{4}{5}x = \frac{33}{5}$$

$$\boxed{5y - 4x = 33}$$ standard form

3. Find an equation of the line through the points $(3,9)$ and $(3,-2)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{9-(-2)}{3-3} = \frac{11}{0}$$ undefined # $\Rightarrow$ Vertical line



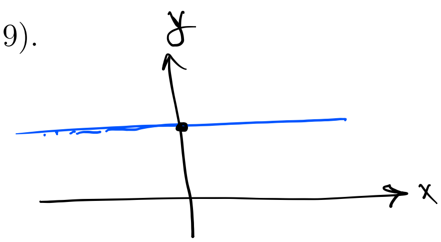
$$\boxed{x = 3}$$

4. Find an equation of the line through the points $(3,9)$ and $(-1,9)$.

$$m = \frac{9-9}{3-(-1)} = \frac{0}{4} = 0$$

$\Downarrow$

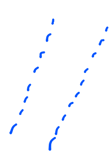
straight line



$$\boxed{y = 9}$$

5. Write an equation of a line a) parallel to and b) perpendicular to the line $5 + x - 2y = 0$ and passing through the point $(4,-3)$ in slope-intercept form.

a) parallel



$\Rightarrow$ same slope

$$y = \overset{\text{slope}}{\overset{\downarrow}{m}}x + b$$

$$5+x = 2y \xrightarrow{\div 2} y = \frac{5}{2} + \frac{x}{2}$$

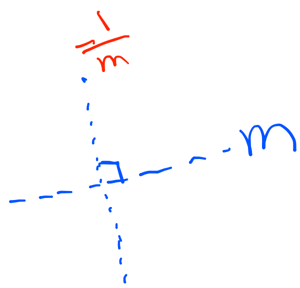
$$\boxed{y = \frac{x}{2} + \frac{5}{2}} \Longrightarrow \text{slope } m = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{1}{2}(x - 4) \rightarrow y + 3 = \frac{1}{2}x - 2$$

$$\boxed{y = \frac{1}{2}x - 5}$$

b) Perpendicular



$$m = -2$$

$$y - (-3) = -2(x - 4)$$

$$y + 3 = -2x + 8$$

$$\boxed{y = -2x + 5}$$

6. Solve the following inequalities. Graph their solution set.

(a) $\frac{x}{2} - \frac{3}{5} \le \frac{1-2x}{10}$

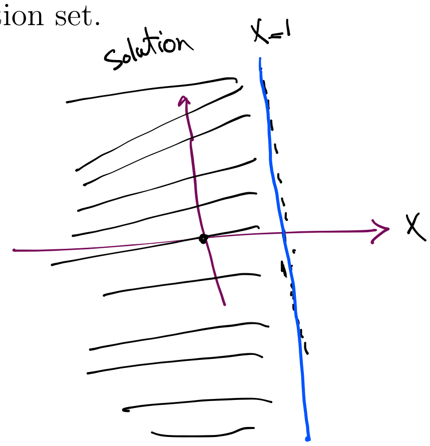
$\frac{x}{2} - \frac{3}{5} \le \frac{1}{10} - \frac{x}{5}$

$\frac{5}{5} \times \frac{x}{2} + \frac{x}{5} \times \frac{2}{2} \le \frac{1}{10} + \frac{3}{5}$

$\frac{5x+2x}{10} \le \frac{1+6}{10}$

$0 \le 1$

$\frac{7x}{10} \le \frac{7}{10} \longrightarrow \boxed{7x \le 7} \longrightarrow \boxed{x \le 1}$



$x = 1$

(b) $-5 \le \frac{1-4x}{2} < 7$

$-5 \le \frac{1}{2} - 2x < 7$

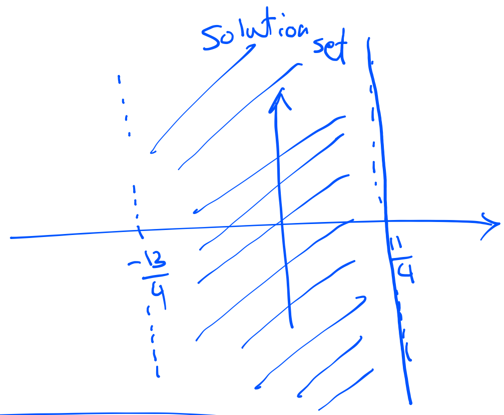
$-\frac{1}{2}$

$-5 - \frac{1}{2} \le -2x < 7 - \frac{1}{2}$

$\frac{-11}{2} \le -2x < \frac{13}{2}$

$\div(-2)$

$\frac{+11}{+4} \ge x > \frac{13}{-4} \Longrightarrow \boxed{-\frac{13}{4} < x \le \frac{11}{4}}$



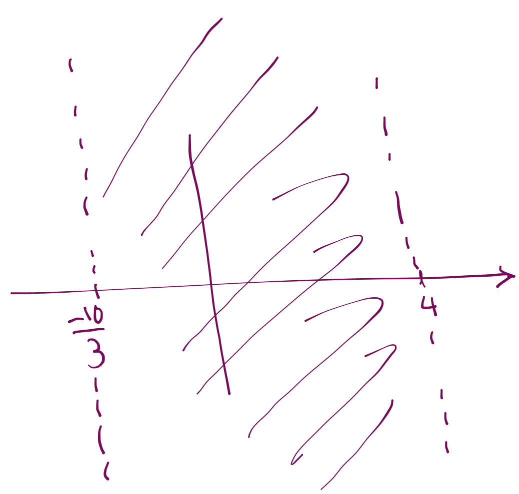
(c) $|3x - 1| < 11$

$\Updownarrow$

$-11 < (3x-1) < 11$

$-10 < 3x < 12$

$\boxed{\frac{-10}{3} < x < 4}$



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7. Simplify the following expression. Write your answer so that each variable appears at most once, and all exponents are positive.

$$\frac{\cancel{15}^{3}(xy^{-1})^2(x^{-2}y^2)^3}{\cancel{5}(x^{-1/2})^4(xy^{-3})^{-2}}$$

$$= \frac{3\ x^2\ y^{-2} \cdot (x^{-2})^3 (y^2)^3}{x^{-2}\ (x)^{-2} \cdot (y^{-3})^{-2}} = \frac{3x^2y^{-2}x^{-6}y^6}{x^{-2}x^{-2}y^{-6}} = \frac{3 \cdot \cancel{x^{-4}} \cdot \cancel{y^4}}{\cancel{x^{-4}} \cdot y^{\cancel{6}\,2}} = \boxed{\frac{3}{y^2}}$$

8. Simplify each radical expression.

(a) $\sqrt[3]{\dfrac{16x^4\cancel{y^2}z^4}{-27x^2y^{\cancel{5}\,3}}}$

$$= \sqrt[3]{\frac{16\ x^2\ z^4}{-27\ y^3}} = \sqrt[3]{\frac{2^3 \times 2\ x^2\ z^3 \cdot z}{(-3)^3\ y^3}}$$

$$= \frac{2z}{-3y}\sqrt[3]{2x^2z}$$

(b) $\sqrt{x^3} + \sqrt{4x^3} - \sqrt{8x}$

$\sqrt{x^2} = |x|$

$$= \sqrt{x^2 x} + \sqrt{2^2 x^2 \cdot x} - \sqrt{4 \cdot 2x}$$

$$= |x|\sqrt{x} + 2|x|\sqrt{x} - 2\underbrace{\sqrt{2x}}_{\sqrt{2}\sqrt{x}}$$

$$= \sqrt{x}\left(|x| + 2|x| - 2\sqrt{2}\right) = \sqrt{x}\left(3|x| - 2\sqrt{2}\right)$$

9. Rationalize the denominator.

(a) $\dfrac{5-z}{\sqrt{5}+\sqrt{z}} \cdot \dfrac{\sqrt{5}-\sqrt{z}}{\underbrace{\sqrt{5}-\sqrt{z}}_{=1}} = \dfrac{(5-z)(\sqrt{5}-\sqrt{z})}{(\sqrt{5})^2-(\sqrt{z})^2}$

Conjugate

$(a+b)\cdot(a-b) = a^2-b^2$

$= \dfrac{\cancel{(5-z)}(\sqrt{5}-\sqrt{z})}{\cancel{5-z}}$ $\boxed{= \sqrt{5}-\sqrt{z}}$

$= (5)^{\frac{1}{2}} - (z)^{\frac{1}{2}}$

(b) $\dfrac{5\sqrt{3}-3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}} \cdot \dfrac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}$

$= \dfrac{(5\sqrt{3}-3\sqrt{2})(2\sqrt{3}-3\sqrt{2})}{(2\sqrt{3})^2-(3\sqrt{2})^2} = \dfrac{(5\sqrt{3})(2\sqrt{3})+(5\sqrt{3})(-3\sqrt{2})-(3\sqrt{2})(2\sqrt{3})+(3\sqrt{2})^2}{4\times 3 - 9\times 2}$

$= \dfrac{30-15\sqrt{6}-6\sqrt{6}+18}{12-18} = \dfrac{48-21\sqrt{6}}{-6}$

$\boxed{= -8 + \dfrac{7\sqrt{6}}{2}}$

10. Simplify the following expression. Leave answer with rational exponents.

$$\left(\frac{a^{5/4}\cdot a^{-1/8}}{a^{1/4}}\right)^{8/3} = \left(\frac{a^{\frac{5}{4}}}{a^{\frac{1}{4}}}\cdot a^{-\frac{1}{8}}\right)^{\frac{8}{3}}$$

$= \left(a^{\frac{5}{4}-\frac{1}{4}}\cdot a^{-\frac{1}{8}}\right)^{\frac{8}{3}}$

$= \left(a^{\frac{4}{4}}\cdot a^{-\frac{1}{8}}\right)^{\frac{8}{3}}$

$= \left(\underbrace{a^{1}}_{a^{\frac{8}{8}}}\cdot a^{-\frac{1}{8}}\right)^{\frac{8}{3}} = \left(a^{\frac{8-1}{8}}\right)^{\frac{8}{3}}$

$= a^{\frac{7}{\cancel{8}}\cdot\frac{\cancel{8}}{3}} = a^{\frac{7}{3}}$

11. Factor each expression.

(a) $x^2y^2 - 10(xy) + 25$ $= (xy-5)^2$

$\underbrace{x^2y^2}_{(xy)^2}$

$t = xy$ $\to$ $t^2 - 10t + 25 = (t-5)^2$

$= (t-5)(t-5)$

(b) $4y^2 - 4y - 3$ $\longrightarrow (2y \quad)(2y \quad)$

$= 4\left[y^2 - y - \frac{3}{4}\right]$

$= 4\,(y - \frac{3}{2})(y + \frac{1}{2}) = 2(y - \frac{3}{2})\,2(y + \frac{1}{2})$

$= (2y-3)(2y+1)$

(c) $(x+2)(x^2-8) + (x+2)^2(x-1)$

$(x+2)\Big((x^2-8) + (x+2)(x-1)\Big) = (x+2)\left[(x^2-8) + \underbrace{x^2 + x - 2}\right]$

$= (x+2)(\underbrace{2x^2 + x - 10}_{(2x+5)(x-2)}) = (x+2)(x-2)(2x+5)$

12. Find the domain of each expression.

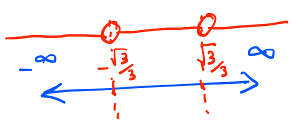
(a) $f(x) = x^2 - 10x + 18$

Poly. $\to$ $\mathbb{R}$ $\quad (-\infty, \infty)$

(b) $\dfrac{7x+1}{9x^2-3}$ $\quad 9x^2 - 3 \neq 0 \quad 9x^2 \neq 3 \to x^2 \neq \frac{1}{3}$

$x \neq \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

$x \neq \pm\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$



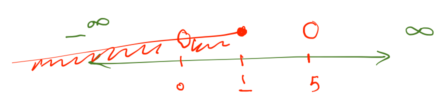
$x \in \left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$

(c) $\dfrac{\sqrt{1-2x}}{x^2-5x}$

$x^2 - 5x \neq 0$
$x(x-5) \neq 0 \to \boxed{x \neq 0}$
$x - 5 \neq 0 \to \boxed{x \neq 5}$

$\to x^2 - 5x \neq 0 \sim x(x-5) \neq 0 \to \boxed{x \neq 0}\ \boxed{x \neq 5}$

$1 - 2x \geq 0 \Rightarrow 1 \geq 2x \to \boxed{x \leq \frac{1}{2}}$



$x \in (-\infty, 0) \cup (0, \frac{1}{2}]$

13. Perform the operations and simplify.

(a) $\dfrac{2x^2-5x-3}{6x^2+3x}\cdot\dfrac{3x^2+12x-15}{x^2+2x-15}$

$3(x^2+4x-5)$

$(x+5)(x-1)$

$3x(2x+1)$ $(x+5)(x-3)$

$2(x^2-\frac{5}{2}x-\frac{3}{2})$

$$= \frac{2(x-1)(x-\frac{3}{2})\cdot \cancel{3}(\cancel{x+5})(x-1)}{\cancel{3}x(2x+1)(\cancel{x+5})(x+3)} = \frac{2(x-1)^2(x-\frac{3}{2})}{x(2x+1)(x+3)}$$

2

(b) $\dfrac{x^2+5x-14}{x^2+8x+7}\div\dfrac{x^2-x-2}{x-3} = \dfrac{x+5x-14}{x^2+8x+7}\cdot\dfrac{x-3}{x^2-x-2}$

$$= \frac{(\cancel{x+7})(x+2)}{(\cancel{x+7})(x+1)}\cdot\frac{(x-3)}{(x-2)(x+1)} = \frac{(x+2)(x-3)}{(x+1)^2(x-2)}$$

(c) $\dfrac{x+2}{x^2-2x-8}-\dfrac{x-2}{x^2-4} = \dfrac{1}{x-4} - \dfrac{1}{x+2}$

$(x-4)(x+2)$ $(x-2)(x+2)$

$$= \frac{(x+2)-(x-4)}{(x-4)(x+2)} = \frac{\cancel{x}+2-\cancel{x}+4}{(x-4)(x+2)} = \frac{6}{(x-4)(x+2)}$$

(d) $\dfrac{\frac{2x}{2x}\cdot\frac{1}{x} - \frac{1}{2x^2}}{\frac{2}{x} - 1\cdot\frac{x}{x}} = \dfrac{\frac{2x - 1}{2x^2}}{\frac{2-x}{x}} =$

$\dfrac{2x-1}{2x^{\cancel{2}}} \cdot \dfrac{\cancel{x}}{2-x} = \dfrac{(2x-1)}{2x(2-x)}$

14. Determine whether the function is even, odd, or neither. Then describe the symmetry.

(a) $f(x) = \dfrac{x(x^2-1)}{5x^4+1}$

Check Next week's Solutions

(b) $g(x) = \sqrt[3]{x^2-1}$

(c) $h(x) = \dfrac{x^3-1}{x^4+2}$

15. If $h(x) = \dfrac{3x}{2} + 1$ evaluate the following:

(a) $h(a)$

Check Next week's Solutions

(b) $h(a+b)$

(c) $\dfrac{h(a+b) - h(a)}{b}$

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