1 Week 14 HOGU: 6.1, Final Exam Review Part 1

Problem 1. Consider the following scenario:

> You want to have \$30,000 in your retirement fund. You currently have \$10,000 to invest for retirement in an account that earns 2.4% ~~quarterly~~ interest, compounded quarterly.

How long would it take for you, in years, to end up with \$30,000 in your account?

TVM Solver
N = ?
I% = 2.4
PV = -10000
PMT = 0
FV = 30000
P/Y = 4
C/Y = 4
PMT: END BEGIN

N = 183.65 quarters
→ 45.9 years
or 46 years

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Problem 2. You are purchasing new furniture from a local retailer. The furniture price is \$12,500, and the retailer offers financing options A and B. After 10 years, if you have not paid anything on the loan, how much interest have you accrued using each loan option?

(a) 9.7% APR, compounded monthly
(b) 9.65% APR, compounded daily

Total owed — Interest
(a) 32845.98 − 12500 = 20345.98
(b) 32871.32 − 12500 = 20321.32

Better!

Which one is the better financing option?

(a) N = 120, I% = 9.7, PV = 12500, PMT = 0, FV = ? → -32845.98, P/Y = 12, C/Y = 12, PMT: END

(b) N = 3650, I% = 9.67, PV = 12500, PMT = 0, FV = ? → -32871.32, P/Y = 365, C/Y = 365, PMT: END

Q: What is the APY for each financing option?

(a) 10.14% APY (b) 10.15% APY

Problem 3. Let $A = \begin{bmatrix} 4 & 2 \\ a & b-c \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 4 & 0 \\ 5 & -8 & 1 \end{bmatrix}$. Only one of the matrix products AC^TB and ACB^T exists. Circle which of the two products exists below.

$\begin{bmatrix} -3 & 5 \\ 4 & -8 \\ 0 & 1 \end{bmatrix}$

AC^TB $\quad$ ACB^T

Then calculate below the matrix product you circled.

ACB^T

$2\times2 \quad 3\times2 \quad 1\times3$

$\times \quad \times$

inner dimensions do not agree!

AC^TB exists!

$2\times2 \quad 2\times3 \quad 3\times1$

$$\begin{bmatrix} 4 & 2 \\ a & b-c \end{bmatrix}\begin{bmatrix} -3 & 4 & 0 \\ 5 & -8 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -12+10 & 16-16 & 0+2 \\ -3a+5(b-c) & 4a-8(b-c) & b-c \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 2 \\ -3a+5b-5c & 4a-8b+8c & b-c \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -2+0+6 \\ -3a+5b-5c+8a-16b+16c+3b-3c \end{bmatrix} = \begin{bmatrix} 4 \\ 5a-8b+8c \end{bmatrix}$$

Problem 4. At Texas A& M, each Math 140 has to pay \$21 for a WebAssign subscription and \$81 for a TI-84 calculator.

Each Math 151 student has to pay \$125 for a WebAssign subscription and \$81 for a TI-84 calculator.

Each Math 251 student has to pay \$100 for a WebAssign subscription and \$0 for a TI-84 calculator. (They do not need one.)

(a) Set up the information above in a ~~2 × 2~~ 2x3 matrix A, labeling each row and column with the information given. Use the column labels: "140" for Math-140 students, "151" for Math-151 students, "WA" for WebAssign subscriptions, and "TI" for TI-84 calculators.

	140	151	251
WA (\$/student)	21	125	100
TI (\$/student)	81	81	0

(b) In the fall semester, 4100 students enroll in Math 140, ~~while~~ 3200 students enroll in Math 151. Set this information up as a ~~2 × 1~~ 3x1 matrix B, labeling each row and column with the information given.

+1500 students enroll in Math 251.

	Students enrolled
140	4100
151	3200
251	1500

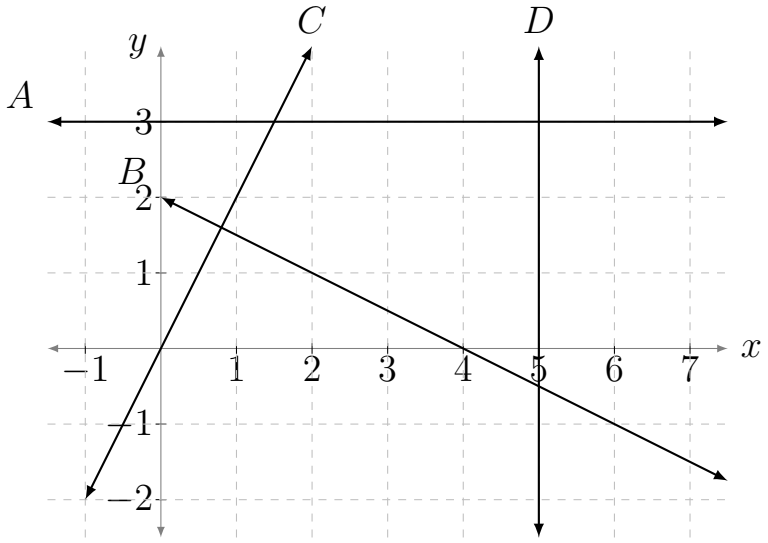
(c) Calculate AB. What is the meaning of each entry in the product matrix AB?

$$\begin{bmatrix} 21\cdot 4100 + 125\cdot 3200 + 100\cdot 1500 \\ 81\cdot 4100 + 81\cdot 3200 + 0\cdot 1500 \end{bmatrix} = \begin{bmatrix} 636100 \\ 591300 \end{bmatrix}$$

\$636100 = amount of money students in 140, 151, 251 spend on WebAssign

\$591300 = amount of money students in 140, 151, 251 spend on TI-84s

Problem 5. Write the equations for each of the lines A, B, C, and D in slope-intercept form. Use fractions, not decimals, in your answers.



A: horizontal line: slope = 0

y-intercept: (0,3)

$y = 3$

B: two points (0,2) + (4,0) (y-intercept)

slope: $\frac{0-2}{4-0} = \frac{-2}{4} = -\frac{1}{2}$

slope-intercept form

$y = -\frac{1}{2}x + 2$

C: two points (0,0) + (1,2)

slope: $\frac{2-0}{1-0} = \frac{2}{1} = 2$

slope-intercept form

$y = 2x$

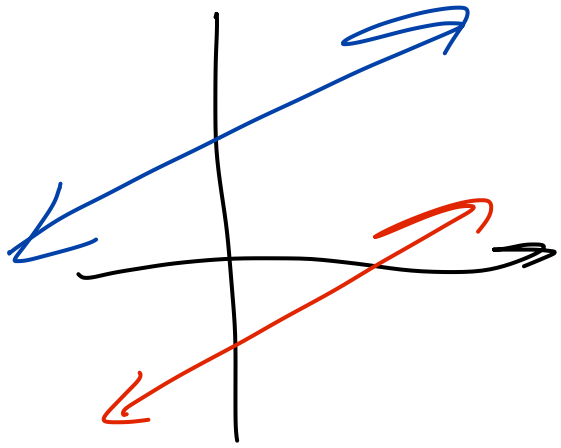
D: vertical line: slope undefined!

$x = 5$

Problem 6. For what value of k does the system of linear equations

$$6x - ky = 24$$
$$-2x + 8y = 24$$

have no solution?



Lines must be parallel!

Same slope.

Inconsistent system: no solution

*Start by finding slope of each line.

$6x - ky = 24 \longrightarrow 6x - 24 = ky \rightarrow y = \frac{6}{k}x - \frac{24}{k}$

$-24 \quad +ky \quad -24 \quad +ky$

slope: $\frac{6}{k}$

$-2x + 8y = 24 \longrightarrow 8y = 2x + 24 \rightarrow y = \frac{1}{4}x + 3$

$+2x \qquad +2x$

slope: $\frac{1}{4}$

equality: $\frac{6}{k} = \frac{1}{4} \rightarrow \boxed{k = 24}$

NOT the same

$6x - 24y = 24$

$-2x + 8y = 24$

Problem 7. Use the RREF function in your calculator to calculate *all* solutions to the system of linear equations.

$$\begin{cases} 4x - y + z = 5 \\ 2y + 6z = 30 \\ x + z = 5 \end{cases}$$

$$\begin{bmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & 6 & 30 \\ 1 & 0 & 1 & 5 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Zero rows & no contradictions → Infinitely many solutions!
(0 = 1)

Convert into system: let $z = t$

$$\begin{aligned} x + z &= 5 & x &= 5 - z = 5 - t \\ y + 3z &= 15 \longrightarrow & y &= 15 - 3z = 15 - 3t \\ z &= t & z &= t \end{aligned}$$

$$\boxed{(5-t, 15-3t, t)}$$

for t a real number

Problem 8. The corner points of a bounded feasible region in quadrant I are $(8, 0)$, $(0, 10)$, $(6, 2)$, and $(3, 4)$. What are the maximum and minimum values of $P = 4x + y$ on this feasible region?

Corner Points	$P = 4x + y$
$(8, 0)$	$P = 4(8) + (0) = 32$
$(0, 10)$	$P = 4(0) + (10) = 10$
$(6, 2)$	$P = 4(6) + (2) = 26$
$(3, 4)$	$P = 4(3) + (4) = 16$

maximum of $P = 32$ @ $(8, 0)$
minimum of $P = 10$ @ $(0, 10)$

Problem 9. Is the given simplex tableau in final form? If it is in final form, state the answer. If not, identify the pivot element.

x	y	z	s_1	s_2	s_3	P	constant	
1	2	0	1	0	0	0	28	28/2
2	8	4	0	1	0	0	16	16/8
0	−1	1	0	0	1	0	0	0/−1
−2	−5	−3	0	0	0	1	0	

Bottom row:

Most negative entry $= -5$ column 2

Ratio $\frac{\text{constant}}{\text{column 2}}$:

Least positive entry (divide by positive numbers ONLY) : $\frac{28}{2}$ row 1

Pivot on "2" in row 1, column 2

Problem 10. (a) *Set up but do not solve* the following linear programming problem:

They also want to sell at least twice as many general gym passes as climbing passes.

The Texas A&M Rec Center has a rock climbing pass that sells for \$50 a month and a general gym pass that sells for \$80 a month. The rec center calculates that every climbing pass sold requires 1 expert employee and 3 novice employees to be on duty, and that every general gym pass requires 2 expert employees and 4 novice employees to be on duty. If there are 32 expert employees and 84 novice employees ready to be put on duty, how many of each type of pass should the Rec Center be selling to maximize their revenue?

Variables: g - number of general gym passes the Rec Center sells

c - number of climbing gym passes the Rec Center sells

R - revenue, in dollars, the Rec Center makes from selling general + climbing gym passes

Objective: Maximize $R = 50c + 80g$

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(b) Write this system of equations in a simplex tableau. What is the first pivot element?

Subject to:

$c + 2g \leq 32$ (expert employees)

$3c + 4g \leq 84$ (novice employees)

$g \geq 2c$ ("twice as many...")

$c, g \geq 0$ (non-negativity constraints)

c	g	s_1	s_2	s_3	R	
1	2	1	0	0	0	32
3	4	0	1	0	0	84
−2	1	0	0	1	0	0
−50	−80	0	0	0	1	0

$\frac{32}{2}$, $\frac{84}{4}$, $\frac{0}{1}$

The "1" in row 3, column 2