Math 150 - Week-In-Review 4

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Problem Statements

1. Find the indicated information for the following functions.

(a) $f(x) = \dfrac{2x^2 - 7x + 3}{x^2 - 2x - 3}$ $= \dfrac{(2x-1)(x-3)}{(x-3)(x+1)}$

Domain: $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

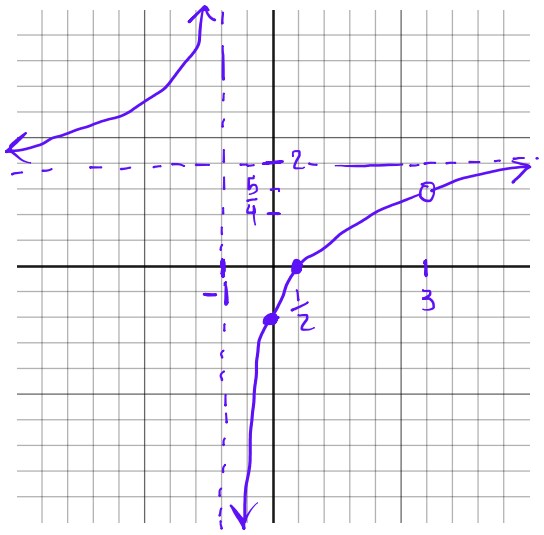
Hole(s): $(3, \frac{5}{4})$

Vertical Asymptote(s): $x = -1$

y-intercept: $(0, -1)$

x-intercept(s): $(\frac{1}{2}, 0)$, ~~$(3, 0)$~~ not in dom.

Horizontal Asymptote(s): $y = 2$



Work:

Hole at $x = 3 \longrightarrow \dfrac{6-1}{3+1} = \dfrac{5}{4}$ $\quad (3, \frac{5}{4})$

y-int. let $x = 0 \longrightarrow \dfrac{2(0)-1}{0+1} = \dfrac{-1}{1} = -1$

x-int let $y = 0 \longrightarrow (2x-1)(x-3) = 0 \longrightarrow x = 3$ & $x = \frac{1}{2}$

Vertical asy. $x = -1 \longrightarrow$ odd multiplicity $\longrightarrow$  or

Check if $x \longrightarrow -1^-$ (from left) e.g. $-1.001 \Rightarrow \dfrac{2(-1.001)-1}{-1.001+1} = \dfrac{-}{-} > 0 \Rightarrow y \to +\infty$

if $x \longrightarrow -1^+$ (from right) e.g. $-0.999 \Rightarrow \dfrac{2(-0.999)-1}{-0.999+1} = \dfrac{-}{+} < 0 \Rightarrow y \to -\infty$

Horizontal asy.

$$\frac{2x-1}{x+1} \overset{\text{end behavior}}{\approx} \frac{2x}{x} = 2 \Rightarrow \boxed{y=2}$$

(b) $g(x) = \dfrac{5x(x-8)}{(x-4)^2}$

Domain: $(-\infty, 4) \cup (4, \infty)$

Hole(s): none

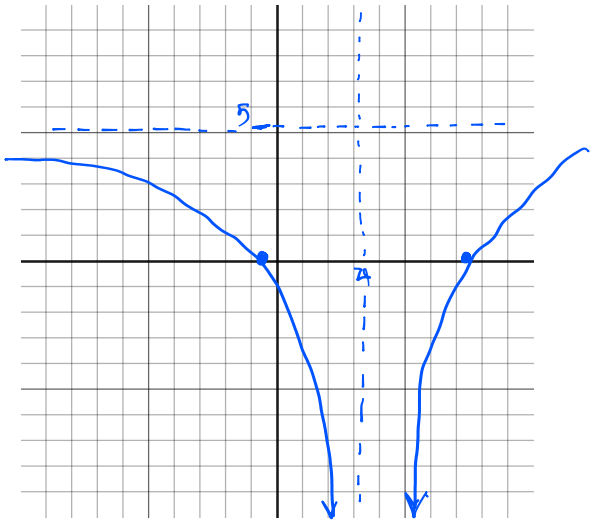
even multiplicity ↑↑ or ↓↓ ← Vertical Asymptote(s): $x = 4$

y-intercept: $(0, 0)$

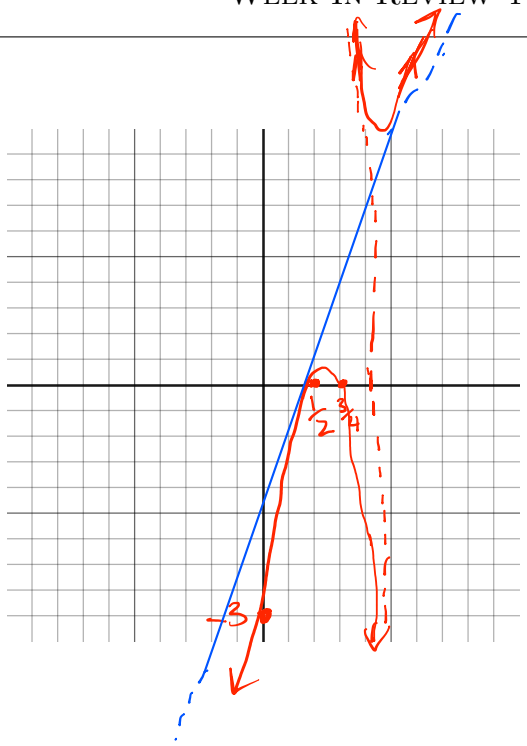
odd multip. cross ← x-intercept(s): $(0,0)$ & $(8,0)$

Horizontal Asymptote(s): $y = 5$

leading terms: $\dfrac{5x^2}{x^2} = 5$



(c) $g(x) = \dfrac{8x^2 - 10x + 3}{x - 1}$ = $\dfrac{(2x-1)(4x-3)}{x-1}$



Domain: $(-\infty, 1) \cup (1, \infty)$

Hole(s): None

Vertical Asymptote(s): $x = 1$

y-intercept: $(0, -3)$

x-intercept(s): $(\frac{1}{2}, 0)$, $(\frac{3}{4}, 0)$

Horizontal Asymptote(s): None

Slant Asymptote: $y = 8x - 2$

Horizontal Asy.:

$$g(x) \simeq \frac{8x^2}{x} = 8x \Rightarrow \text{None!}$$

Slant Asy. ?

$$\begin{array}{r|l} & 8x - 2 \\ \hline x-1 & 8x^2 - 10x + 3 \\ & 8x^2 - 8x \\ \hline & -2x + 3 \\ & -2x + 2 \\ \hline & 1 \end{array}$$

$$\Rightarrow \frac{8x^2 - 10x + 3}{x - 1} = 8x - 2 + \frac{1}{x - 1}$$

$x = 1$ V. A.

$x \to 1^+$ $\quad \frac{+}{+} > 0$ $\quad g(x) \to +\infty$

$x \to 1^-$ $\quad \frac{+}{-} < 0$ $\quad g(x) \to -\infty$

2. $g(x) = \dfrac{x-2}{x^2+4}$ → irreducible

Domain: $\mathbb{R}$

Hole(s): None

Vertical Asymptote(s): none

y-intercept: $(0, -\frac{1}{2})$

x-intercept(s): $(2, 0)$

Horizontal Asymptote(s): $y = 0$