Note ♯7: Exam 03 review

Problem 1. Calculate the double integral.

$$\iint_R 2xye^{xy^2}dA, \quad R = [0,1] \times [0,2]$$

Problem 2. Evaluate the iterated integral.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} e^{x^2+y^2} dydx$$

Problem 3. Sketch the region of integration and evaluate the integral.

$$\int_0^1 \int_{2\sqrt{x}}^2 \sqrt{y^3+1}dydx$$

Problem 4. Find the volume of the solid that is enclosed by the paraboloids $x = y^2 + z^2$ and $x = 3 - 2y^2 - 2z^2$.

Problem 5. Express the integral $\iiint_E f(x,y,z)dV$ as an iterated integral in the order $dydzdx$, where E is the solid bounded by the given surfaces.

$$y = x^2, \quad z = 0, \quad 2y + z = 2$$

Problem 6. Find the volume of the solid tetrehedron with vertices $(0,0,0), (1,0,0), (0,1,0)$ and $(0,0,1)$ using triple integral.

Problem 7. (a) Change $(2, -2\sqrt{3}, 5)$ from rectangular to cylindrical coordinates.
(b) Change $(\sqrt{3}, -1, -2\sqrt{3})$ from rectangular to spherical coordinates.

Problem 8. Find the volume of the solid below the cone $z = \sqrt{4x^2 + 4y^2}$ and above the ring $4 \leq x^2 + y^2 \leq 9$, where the ring is in the xy-plane and to the left of the y-axis.

Problem 9. Compute $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)\, dzdydx$.

Problem 10. Evaluate $\iiint_E \sqrt{x^2+y^2+z^2}dV$, where E lies above the cone $z = \sqrt{3(x^2+y^2)}$ and between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$.

Problem 11. Find the absolute value of the Jacobian of the transformation $x = u^2 + uv, \quad y = uv^2$ evaluated at $u = 1, v = -2$.

Problem 12. Use the transformation $u = x - y, v = x + y$ to rewrite

$$\iint_R x^2 - y^2 dA,$$

where R is the square with vertices $(2,0), (0,2), (-2,0)$, and $(0,-2)$. Do not evaluate the integral.

Problem 13. Use the given transformation to evaluate the integral

$$\iint_R (x+2y)e^{x^2-4y^2}dA$$

where R is the parallelogram enclosed by the lines $x + 2y = 0, x + 2y = 5, x - 2y = 0$, and $x - 2y = 4; u = x + 2y, v = x - 2y$.