

TEXAS A&M UNIVERSITY College of Arts & Sciences

Problem 1. A company that makes calculators has a cost function given by C = 30x + 90,000 dollars and a revenue function given by $R = 300x - \frac{x^2}{30}$ dollars, where x is the number of calculators produced and sold each week. If the number of calculators produced and sold is increasing at a rate of 500 calculators per week, find the rate of change of profit with respect to time when 6000 calculators are produced and sold each week.

Problem 2. State the difference between partition numbers of f' and the critical values of f.

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Problem 3. Two ships leave their respective ports at the same time and are headed to the same destination point. After one hour, ship A is 3 miles south of the distination port, traveling at 6 miles per hour. Ship B is 4 miles west of the destination port, traveling at 7 miles per hour. At what rate is the distance between the two ships changing?

Problem 4. Consider a function f that is continuous on its domain $D : (-\infty, 2) \cup (2, \infty)$ and where $f'(x) = \frac{8(x+4)}{(x-2)^4}$. Find all critical values of f, the intervals where f is increasing or decreasing, and where the local extrema of f occur. Label each type of extrema.

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Problem 5. A car is moving along the graph of $x^2 + y^2 = 25$. When the car is at the point (3, -4) on the graph, the *x*-coordinate is decreasing at a rate of 1.2 units per second. What is the rate of change of the *y*-coordinate at that time?

Problem 6. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. If the radius is decreasing at a rate of 2.5 cms per second, what is the rate of change of the volume of the sphere, when the radius is 8 cms?

Problem 7. Find the critical values of f, intervals where f is increasing or decreasing, and all local extrema of f. Classify each extrema.

(1)
$$f(x) = \frac{3x^2 - 2x}{(x-4)^2}$$

(2)
$$f(x) = x^2 e^{-x}$$

(3)
$$f(x) = 125\ln(x) - \frac{5}{2}x^2$$
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Problem 8. If a function has a critical value at x = 6, does that mean that the function must have a local maxima or a local minima at x = 6?

Problem 9. A boat is being pulled into a dock at a rate of 30 meters per minute by a winch that is located 10 meters above the water.

(1) How fast is the distance between the boat and the dock changing when the boat is 15 meters away from the dock?

(2) At what rate is the boat approaching the dock when the boat is 15 meters away from the dock?

Problem 10. Answer the following questions based on the given graph:



- (1) Where is f(x) < 0?
- (2) Find the partition numbers of f'.
- (3) What are the critical points of f?
- (4) Where is f'(x) < 0?
- (5) Where are the local extrema of f? Classify each extrema.

Problem 11. Answer the following questions based on the given graph:



- (1) Find the partition numbers of f'.
- (2) What are the critical points of f?
- (3) Where is f(x) decreasing?
- (4) Where is f'(x) < 0?
- (5) Where are the local extrema of f? Classify each extrema.
- (6) Where are the local extrema of f'? Classify each extrema.

Problem 12. The length of a 12 foot by 8 foot rectangle is increasing at a rate of 3 feet per second while its width is decreasing at a rate of 2 feet per second.

(1) How fast is the perimeter of the rectangle changing?

(2) What is the rate of change of the area of the rectangle?

Problem 13. A complany that makes deluxe toasters has a weekly demand equation given by $p(x) = 150e^{-0.02x}$, where p(x) is the price per toaster in dollars, when x toasters are demanded.

(1) What is the marginal revenue for the company when 60 to asters are sold? Interpret your answer.

(2) At what value of x is the company's revenue increasing?