EXAM 2 REVIEW OVER CHAPTERS 3 AND 4

- Setting Up Linear Programming Problems
- Graphing Systems of Linear Inequalities in Two Variables
- Graphical Solution of Linear Programming Problems
- Mathematical Experiments
- Basics of Probability
- Rules of Probability
- Probability Distributions and Expected Value

chapter 3 ← lengthy word problems

chapter 4

Pr 1. A local gordita truck has $9000 available each month for advertising. Ads in the university newspaper will cost $500 each, while radio ads costs $30 each, and internet banners on the university library page cost 80 cents each. The ~~taco~~ gordita truck wants to run at least twice as many radio ads as newspaper ads. Approximately 6000 students will see each newspaper ad, 4000 students will hear each radio ad, and 1200 students will see each internet banner. How many of each type of ad should the taco truck run to maximize the number of students who see or hear the ads? **Set up, but do not solve.**

S = the number of students who see the ads

n = the number of newspaper ads

r = the number of radio ads

b = the number of internet banners

Maximize $S = 6000n + 4000r + 1200b$

subject to:

Total amount spent → • $500n + 30r + .80b \leq 9000$ 80 cents = $.8

at least Twice as many radio as newspaper • $r \geq 2n$

(common errors
$2r \geq n$
$r \leq 2n$ } wrong
$r - 2n \geq 0$
or $2n - r \leq 0$)

• $n \geq 0$, $r \geq 0$, $b \geq 0$

Section 3.2

Pr 2. Graph the system of inequalities below. Then determine if the solution set is bounded or unbounded and all corner points of the solution set.

find → Section 3.3

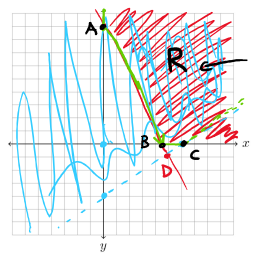
True shading

$2x + y \geq 9$
$2x - 3y < 12$
$x \geq 0,\ y \geq 0$

$2x+y \geq 9$
$2x+y = 9$
Set $x=0$
$2\cdot 0 + y = 9$
$0 + y = 9 \to y = 9$
$(0,9)$

Set $y=0$
$2x + 0 = 9$
$x = \frac{9}{2} = 4.5$ $(4.5,0)$

Test point $(0,0)$
$2\cdot 0 + 0 \geq 9$
$0 \geq 9$ NO



$2x-3y < 12$
$2x-3y = 12$
Set $x=0$
$2\cdot 0 - 3y = 12$
$-3y = 12$
$y = \frac{12}{-3} = -4$
$(0,-4)$

Set $y=0$
$2x - 3\cdot 0 = 12$
$2x - 0 = 12$
$x = \frac{12}{2} = 6$
$(6,0)$

Test point $(0,0)$
$2\cdot 0 - 3\cdot 0 = 0 < 12$ ✓

corner points:
A = $(0,9)$
B = $(4.5, 0)$
C = $(6,0)$

what if the intersection of lines was on the boundary?
To find point D,

$2x + y = 9$
$2x - 3y = 12$
} and solve

$$\left[\begin{array}{cc|c} 2 & 1 & 9 \\ 2 & -3 & 12 \end{array}\right] \xrightarrow{RREF}$$

Pr 3. An investment company has two funds, A and B, that you can pick from for your personal investments. Each unit of fund A costs \$15, yields an annual return of 6%, and has a risk index of 1.5 per unit. Each unit of fund B costs \$12, yields an annual return of 5%, and has a risk index of 0.8 per unit. You have \$36,000 available for investing and want to earn at least \$1,800 in interest in the coming year. How many units of each fund should you purchase in order to meet your goals and also to minimize the total risk index for your portfolio?

A = the number of units of fund A

6% annual return on

B = the number of units of fund B

R = the total risk

Minimize $R = 1.5A + .8B$

Subject to:

$15A + 12B \le 36000$ (amount invested)

$.06 \times 15A + .05 \times 12B \ge 1800$

$A \ge 0 \quad B \ge 0$

$15A + 12B = 36000$

Set $A = 0$ $\quad 12B = 36000$

$B = 3000$

$(0, 3000)$

Set $B = 0$

$15A + 0 = 36000$

$A = 2400$

$(2400, 0)$

$.9A + .6B = 1800$

Set $A = 0$ $\quad .6B = 1800$

$B = 3000$

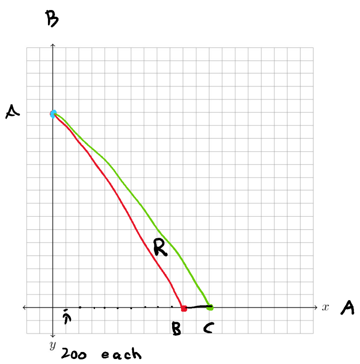
$(0, 3000)$

Set $B = 0$

$.9A = 1800$

$A = 2000$

$(2000, 0)$



Test Point $(0,0)$ True for green, False for red

	Corner Point	risk $R = 1.5A + .8B$
A	$(0, 3000)$	2400 ←
B	$(2400, 0)$	3600
C	$(2000, 0)$	3000

$$\begin{bmatrix} 0 & 3000 \\ 2400 & 0 \\ 2000 & 0 \end{bmatrix} \begin{bmatrix} 1.5 \\ .8 \end{bmatrix}$$

minimum with 0 units of A, 3000 units of B, with a risk of 2400.

what if we achieved our minimum at two corner points? correct: the line segment between the two points.

Pr 4. A 4-H member raises only goats and pigs. She has pen space for no more than 18 animals. She spends \$30 to raise each goat and \$60 to raise each pig and she has \$900 available for this project. Each goat produces \$15 in profit and each pig \$30 in profit. Using linear programming techniques, we find 15 pigs and no goats should be raised to maximize the 4-H member's profit. Are there any leftover resources, and if so what are they?

→ profits are ~~not~~ a constraint

Are there leftover resources

g = number of goats

p = number of pigs

$30g + 60p \le 900$ (raising money)

$g + p \le 18$ (pen space)

$p = 15$ $\quad 30 \cdot 0 + 60 \cdot 15 = 900$ → • no money left over

$g = 0$ $\quad 0 + 15 \le 18 \rightarrow 18 - 15 = 3$ • we have space for 3 more animals

Pr 5. In an experiment, a fair standard 12-sided die is rolled, noting whether or not the number facing is a multiple of 5, and then a card is drawn from a well-shuffled deck. Write the sample space for the experiment.

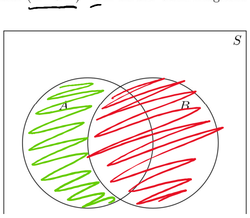
52 card and the suit is noted

multiple of 5 → H, D, S, C

not a multiple of 5 → H, D, S, C

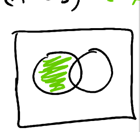
{ (multiple of 5, H), (multiple of 5, D), (mult. of 5, C), (mult. of 5, S), (not mult of 5, H), (not mult of 5, D), (not mult of 5, C), (not mult of 5, S) }

Pr 6. Shade $(A^C \cup B)^C \cup B$ on the Venn Diagram. Your answer must be obvious to the instructor.



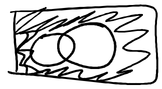
$A^c \cup B$

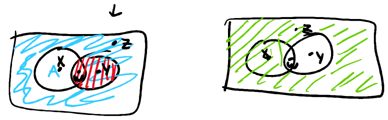
$(A^c \cup B)^c = A \cap B^c$



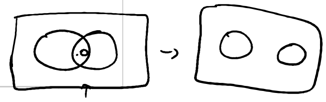
Shade $(A^c \cup B) \cap B^c$

union "or" intersection





$A^c \cup B$



final drawing

mutually exclusive

Pr 7. Assuming two fair standard four-sided dice are rolled, one maroon and one white, and let

A = the event "a sum of 4 is rolled",

B = the event "a 3 is showing on the maroon die", and

D = the event "an number less than 3 shows on the white die".

B^c = "a 3 is not showing on maroon"

(a) Write the symbolic equivalent of the event "a sum of 4 is rolled, but the ~~blue~~ maroon die isn't showing a 3 and the ~~green~~ white die is showing a number less than 3".

and → $A \quad B^c \cap D$

union ← or not correct

intersection

$\boxed{A \cap B^c \cap D}$

sum of 3 and (1,2)

(b) Verbally describe $D^C \cup A^C$

or

" a number at least 3 shows on the white die or the sum is not 4."

"we did not roll a sum of 4" A^c

Pr 8. A survey of 50 veterans from the Air Force and Navy was taken to gather information on their service career and what life is like outside of the miliatry. A breakdown of those surveyed is shown in the table. Suppose a randomly selected veteran from the Air Force or Navy is interviewed. What is the probability the person chosen is

	Air Force	Navy	Total
Private	5	8	**13**
Corporal	11	8	**19**
Sergeant	5	4	**9**
Lieutenant	2	2	**4**
Captain	2	3	**5**
Total	**25**	**25**	**50**

(a) P(is a Corporal or in the Navy) $= \frac{36}{50} = \frac{18}{25}$

2nd row, 2nd column

Approach 1: add 2nd row total + 2nd col total − Navy corporals

$= 19 + 25 - 8 = 36$

$= 25 + 11 = 36$

(b) P(in the Air Force or in the Navy) $= \frac{50}{50}$

(c) P(is a Private and a Lieutenant) $= \frac{0}{50}$

$\emptyset \quad P \cap L = \emptyset$

(d) P(is not in the Air Force, but is a Sergeant) $= \frac{4}{50} = \frac{2}{25}$

and

P(in the Navy and is a Sergeant)

(e) P(is not a Captain and is in the service) =

P(is not a captain)

$= \frac{13+19+9+4}{50} = \frac{50-5}{50} \rightarrow P(A) = 1 - P(A^c)$

$= \frac{45}{50}$

$P(A) = 1 - P(A^c)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Pr 9. Given $P(A) = 0.4$, $P(B) = 0.7$, and $P(A \cup B) = 0.9$, compute $P\left[(A \cap B)^C\right]$.

$$P((A \cap B)^c) = 1 - P(A \cap B)$$
$$= 1 - (P(A) + P(B) - P(A \cup B))$$
$$= 1 - (.4 + .7 - .9)$$
$$= 1 - (.2) = 1 - .2 = \boxed{.8}$$

$(A \cap B)^c = A^c \cup B^c$

$P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c) = P(A^c) + P(B^c) - P((A \cup B)^c)$

get stuck

Pr 10. Your insurance company has a policy to insure personal property. Assume your personal property is worth \$2,500, and according to campus statistics there is a 2% chance that your property will be stolen during the next year and a 12% chance that your property is damaged beyond repair through natural causes during the next year. If your property is stolen the policy will give you \$2,500, while if it is damaged beyond repair you receive get \$1,200. What is the insurance company's expected profit on this policy, if the premium for the policy is \$300?

	stolen	damaged	every thing is fine
$P(x)$	.02	.12	.86
X	-2200	-900	300

$1 - .12 - .02 = .86$

$300 - 1200 = -900$

$300 - 2500$

$E(X) = -2200 \times .02 - 900 \times .12 + 300 \times .86$
$= \$106$

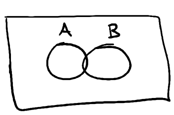
$\boxed{\text{Expected profit is } \$106}$

Alternative question: what should the premium be to expect to break-even

X	$-2500 + p$	$-1200 + p$	p

$$E(X) = (-2500 + p) \times .02 + (-1200 + p) \times .12 + p \times .86$$
$$= -2500 \times .02 - 1200 \times .12 + p = 0$$
$$p = 2500 \times .02 + 1200 \times .12$$
$$= 194$$

4.3 word problem



A = "You went to A+M men's basketball game" on Friday

B = "You went to a soft ball game on Friday.

300. students

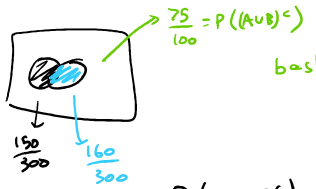
150 went to basketball

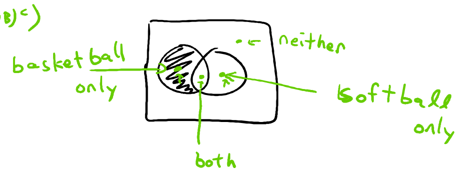
160 went to softball

75 went to neither.

What is the probability that a student went to basketball but not softball?

Draw the venn Diagram $A \cap B^c$





$$P(A \cap B^c) = \underline{P(A) - P(A \cap B)}$$
$$= P(A) - \left(P(A) + P(B) - P(A \cup B) \right)$$
$$= P(A) - P(A) - P(B) + P(A \cup B)$$
$$= \underline{P(A \cup B) - P(B)}$$
$$= 1 - P((A \cup B)^c) - P(B)$$
$$= 1 - \frac{75}{300} - \frac{160}{300}$$
$$= \frac{300 - 75 - 160}{300} = \frac{65}{300}$$

probability $P(A \cap B) = P(A) + P(B) - \underline{P(A \cup B)}$ or

$$= 1 - P((A \cap B)^c) =$$
$$= \frac{150}{300} + \frac{160}{300} - \frac{300-75}{300}$$

$P(A \cup B) = 1 - P((A \cup B)^c)$

$$= \frac{310 - 225}{300} = \frac{85}{300}$$

not mutually exclusive.