Note $\sharp1$: Sections 12.1-12.5

Problem 1. a) Find the center and radius of the sphere $x^2 + y^2 + z^2 - x + 4y - 10z - 1 = 0$. b) Find the equation of the sphere with center (1, 4, 3) that touches the xy plane.

Problem 2. For the picture seen below, write \mathbf{v} in terms of \mathbf{u} and \mathbf{w} .



Problem 3. Given $\mathbf{a} = \langle 1, -1, 5 \rangle$ and $\mathbf{b} = \langle -3, 2, 1 \rangle$,

- a) find a unit vector in the direction of $\mathbf{a} + 2\mathbf{b}$.
- b) find the vector that has the same direction as $\mathbf{a} + 2\mathbf{b}$ but has length 4.

Problem 4. Compute $\mathbf{a} \cdot \mathbf{b}$ if

- a) $\mathbf{a} = \langle 4, 5, -1 \rangle$ and $\mathbf{b} = \langle 2, 1, 3 \rangle$.
- b) $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and $\theta = 120^{\circ}$.
- c) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and \mathbf{a} is perpendicular to \mathbf{b} .
- d) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and \mathbf{a} is parallel to \mathbf{b} .

Problem 5. The points A(0, -1, 6), B(2, 1, -3) and C(5, 4, 2) form a triangle. Find $\angle C$.

Problem 6. Let \mathbf{a} , \mathbf{b} , and \mathbf{c} be three dimensional vectors. Which of the following expressions are meaningful? Which are meaningless?

- a) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$
- b) $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$

Problem 7. Determine whether the given vectors are orthogonal, parallel, or neither.

a) $\mathbf{a} = \langle 3, -1, 2 \rangle$, $\mathbf{b} = \langle 6, -2, 4 \rangle$ b) $\mathbf{a} = \langle 1, 2, -1 \rangle$, $\mathbf{b} = \langle 2, 3, -1 \rangle$ c) $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

Problem 8. Find the scalar and vector projections of (2, 4, 6) onto (1, 3, 5).

Problem 9. Find the cross product of (1, 1, 3) and (-2, -1, -5).

Problem 10. Find $|\mathbf{u} \times \mathbf{v}|$ and determine if $\mathbf{u} \times \mathbf{v}$ points in or out of the page.



Problem 11. Find the area of the parallelogram determined by $\mathbf{a} = \langle 3, 0, 2 \rangle$ and $\mathbf{b} = \langle 1, -4, 5 \rangle$.

Problem 12. Find a vector equation of the line that passes through the point (2, -5, 1) and is parallel to the vector $\langle 8, 10, -7 \rangle$.

Problem 13. Find parametric equations and a symmetric equation for the line passing through the points (-2, 3, 4) and (5, 2, 8).

Problem 14. Find an equation of the plane passing through the point (3, 4, 5) and perpendicular to $\langle -1, 2, 5 \rangle$.

Problem 15. Find an equation of the plane passing through the point (1, 5, -3) and perpendicular to the line x = 2 - 4t, y = 2t, z = -1 + t.

Problem 16. Find the equation of the plane that passes through the points P(1,0,1), Q(2,3,4) and R(2,1,1).

Problem 17. Find an equation of the plane passing through the point (-1, -3, 2) that contains the line x = -1 - 2t, y = 4t, z = 2 + t.

Problem 18. Consider the lines $\mathbf{r}_1(t) = \langle 2+t, 2t, 5+t \rangle$ and $\mathbf{r}_2(s) = \langle s, -4+4s, 3+s \rangle$.

- a) Find the point of intersection of the lines
- b) Find an equation of the plane that contains these lines.

Problem 19. Consider the planes z = x + y and 2x - 5y - z = 1.

- a) Find the angle between the planes.
- b) Find the line of intersection of the planes.