Question 1. Use Laplace transforms to solve the following IVP:

$$x'' - 2x' + 2x = 0, \qquad x(0) = -1, x'(0) = 1.$$

Question 2. Use Laplace Transforms to solve:

$$x'' + 3x' + 2x = e^{-t}, \qquad x(0) = 0, \quad x'(0) = 0.$$

Question 3. Consider the Initial Value Problem:

$$x'' - 2x' + 2x = \begin{cases} 0 & 0 \leq t < 3 \\ (t-3) & 3 \leq t \end{cases}, \qquad x(0) = 1, x'(0) = 0.$$

Find $X(s)$, the Laplace transform of the solution of the IVP.

Question 4. Find the inverse Laplace transform of:

$$X(s) = \frac{(e^{-s} - e^{-2s})}{s^2 - 2s + 2}.$$

Question 5. Solve the following Initial Value Problem:

$$x'' - 2x' + 2x = \delta(t-2), \qquad x(0) = 0, x'(0) = 0.$$

Question 6. Use convolution to solve $x'' + x = \delta(t-2)$ with $x(0) = x'(0) = 0$. You must evaluate the integral in the definition of convolution (i.e. you can't leave it as an unevaluated integral.)

Question 7. Consider the ODE:

$$(1-t)x''(t) + x(t) = 0.$$

Write the form that the solution will take if expanded in a power series at the point $t_0 = 0$ (don't take any steps to find the coefficients $\{a_k\}$, yet.)

Question 8. Plug this series into the ODE to find a recurrence relation for the coefficients $\{a_k\}$.

Question 9. Consider the ODE:

$$(t^2 + 2t + 2)x''(t) + 2x'(t) + x(t) = 0.$$

Find a lower bound on the radius of convergence of the power series representation of the general solution expanded at $t = 0$.

Question 10. Solve $x'' - 2tx' + 2x = 0$ with $x(0) = 0$ and $x'(0) = 1$. HINT: Use series solutions.