Exam 1 Review: Covering sections 5.5-7.1

PLEASE SCAN THE QR CODE BELOW

We will begin at 7PM. A problem will be displayed on the table monitors. Collaborate with your table on how to solve each problem. If you have a question, raise your hand. At the end of a predetermined number of minutes, the solutions will be displayed on the wall monitors. Feel free to take a picture of the solution, as the solutions are not posted.

In order to get through content, for some of the volume, area, and water pumping problems we will just set up the integral. But on the common exam, some of them you will be expected to evaluate.

This non solved problem set is posted at https://mlc.tamu.edu/Help-Services/Hands-on,-Grades-up.

Integration by u-sub: Discuss with your table how to know when to use this technique.

Integration by parts: Use the acronym 'ILATE'. Discuss with your table how this acronym works.

$$\int u\,dv = uv - \int v\,du$$

I (Inverse trig) : $\int \arcsin(kx)\,dx$, $\int \arccos(kx)\,dx$, $\int \arctan(kx)\,dx$. Let $u =$ inverse trig part.

L (Logarithm) : $\int x^n \ln(kx^m)\,dx$, $\int x^n(\ln(x))^m\,dx$. Let $u =$ entire logarithm part.

A (Algebraic, and n MUST be a positive integer) : $\int x^n e^{kx}\,dx$, $\int x^n \sin(kx)\,dx$, $\int x^n \cos(kx)\,dx$. Let $u = x^n$ (**Note**: kx must be degree one and n must be a positive integer). The short cut of tabular can be used in this instance only, and both techniques will be shown on solutions.

These last two are the 'loops' which are integrals of the product of e^x and $\sin x$ or product of e^x and $\cos x$. This requires two iterations of parts and can be done in either order. Choose u however you want, as long as on the second iteration, you choose u as you did in the first iteration.

T (Trig) and **E** (Exponential): $\int e^{kx}\cos(nx)\,dx$, $\int e^{kx}\sin(nx)\,dx$.
Let $u = e^{kx}$ OR $u =$ sine/cosine.

Problem 1. $\int_0^2 xe^{3x^2}\,dx$

Hint: Does u-sub work? If not, is it eligible for parts?

Problem 2. $\displaystyle\int \frac{x}{\sqrt{16+x}}\,dx$

Does u-sub work? If not, is it eligible for parts?

Problem 3. $\int_0^2 \frac{x}{e^{2x}}\,dx$

Does u-sub work? If not, is it eligible for parts?

Problem 4. $\displaystyle\int_0^{\pi/2} x^2 \sin x \, dx$

Does u-sub work? If not, is it eligible for parts?

Problem 5. $\int x \ln(2x)\, dx$

Does u-sub work? If not, is it eligible for parts?

Problem 6. $\int (\ln x)^2 \, dx$

Does u-sub work? If not, is it eligible for parts?

Problem 7. $\int x^5 \cos(2x^3)\, dx$

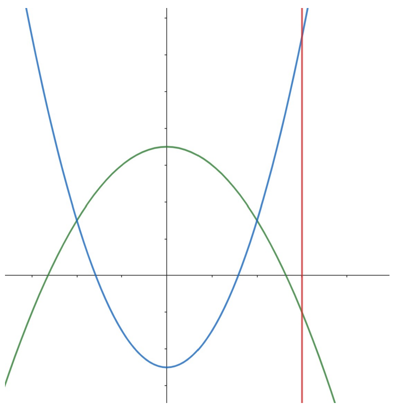
HINT: Notice the power on the angle of cosine is not one! Do a u-sub first.

Area

Problem 8. Set up an integral or integrals that gives the area bounded by $x = 3y - y^2$ and $y = -\frac{x}{2}$.

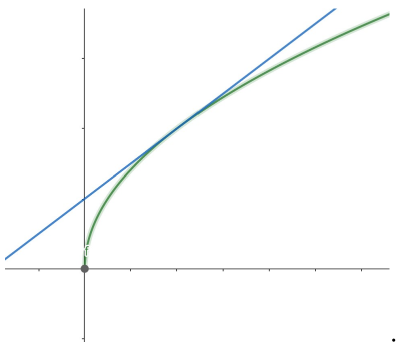
Problem 9. Set up an integral or integrals that gives the area bounded by $y = 7 - x^2$ and $y = 2x^2 - 5$, $x = -2$, $x = 3$.

Hint: Find where the graphs intersect. Does $x = 3$ fall between the intersection points? Sketch the bounded region.



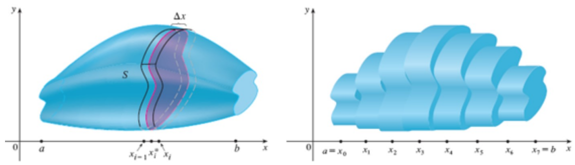
Problem 10. Set up an integral or integrals that gives the area bounded by by $y = \sqrt{x}$, the tangent line to $y = \sqrt{x}$ at the point $(4, 2)$ and the y axis.

Hint: Find the tangent line and shade the region bounded by $y = \sqrt{x}$, the y-axis and the tangent line to $y = \sqrt{x}$ at the point $(4, 2)$.



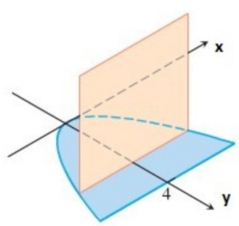
Volume

• Method of slicing: This method is used if the area of an arbitrary cross section is known. **This includes the method of disks and washers**.

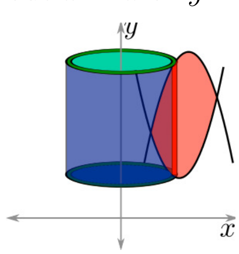


$V = \int_a^b A(x)\,dx$ where $A(x)$ = area of an arbitrary cross-section perpendicular to the x-axis, as illustrated above.

$V = \int_c^d A(y)\,dy$ where $A(y)$ = area of an arbitrary cross-section perpendicular to the y-axis, as illustrated below.



• Shells: This is an illustration of volume using cylindical shells where we rotate a bounded region R around the y axis. This is not volume by slicing, rather we rotate an arbitrary rectangle within R parallel to the axis of rotation about the y-axis. Here, $V = \int_a^b 2\pi x h\,dx$, where x is the radius of the shell and h = height of shell, which in this sketch, h is Top function - Bottom. Using shells where R is rotated about the y-axis, $V = \int_c^d 2\pi y h\,dy$ where h = Right-Left. Rotation about $x = a$ or $y = c$ affects the radius.



HOW DO YOU KNOW WHAT METHOD TO USE??: It depends, disuss it with your table as we proceed with problems.

Problem 11. Consider the region R bounded by $y = x^2 - 4x$ and $y = 0$. Find the volume of the solid obtained by revolving R about the y-axis. Solve the integral.

Hint: Sketch the region R. What are the x intercepts? Where is the vertex? Is the bounded region above or below the x axis?

Problem 12. Consider the region R bounded by $x = y^2$ and $x = 4$. Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating R about the line $x = 4$ using:

a.) The disk method

b.) The shell method

Problem 13. Consider the region R bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$. Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating R about the line $y = -2$ using

a.) The method of washers

b.) The method of shells

Problem 14. Find the volume of the solid whose base is the triangular region with vertices $(1, 1)$, $(2, 2)$ and $(3, 1)$. Cross-sections perpendicular to the y-axis are semi-circles. Set up integral only. Do not evaluate.

Hint: Sketch the triangular base. If you take an arbitrary cross-section perpendicular to the y axis, what is the area of an arbitrary cross-section? What is the variable of integration?

Problem 15. Find the volume of the solid S whose base is bounded by the region $4x^2+25y^2 = 1$, and cross-sections perpendicular to the x-axis are squares.

Hint: Sketch the elliptical base. If you take an arbitrary cross-section perpendicular to the x axis, what is the area of an arbitrary cross-section? What is the variable of integration?

Work

Spring Problems. Hooke's law says the **force** required to maintain a spring stretched x units beyond its natural length is $f(x) = kx$. To find the work to stretch the spring, integrate the force function $f(x) = kx$. Limits of integration depend on how far beyond the natural length you are stretching the spring.

Problem 16. A spring has a natural length of 3 m. The force required to hold a spring stretched to a length of 7 m is 5 N. Find the spring constant, k.

Problem 17. A spring has a natural length of 1 foot. If the work required to stretch the spring from 3 feet to 4 feet is 10 foot pounds:

a.) Find the spring constant, k.

b.) Find the work done in stretching the spring from its natural length to 2 feet beyond its natural length.

c.) What is the force required to hold the spring to 5 feet?

Rope Pulling: Discuss with your table how to find the variable force, $f(x)$, then integrate to find work.

Problem 18. A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 feet deep. Find the work done.

Problem 19. A rope that weighs 200 N and is 100 meters long hangs vertically from the top of a surface. How much work is done in pulling the first 10 meters of the rope to the top of the surface?

Problem 20. A rectangular tank 10 m long, 3 m wide and 5 m is deep filled with water. Set up but do do not evaluate an integral that gives the work required to pump the top 2 meters of water to the top of the tank. Clearly indicate on a vertical axis where you place the origin and which direction is positive. The weight density of water is $\rho g = 9800 N/m^3$.

Problem 21. Consider the region R in the first quadrant bounded by $y = x^2$, the y-axis, and the line $y = 9$. Rotate R about the y-axis. This creates a bowl shaped tank, and we may assume the tank is closed at the top. Measure length in meters. If this tank is filled with water to a depth of 5 meters, set up but do not evaluate an integral that gives the work done in pumping the water through a 1 meter spout that is located at the top of the tank. Clearly indicate on a vertical axis where you place the origin and which direction is positive. The weight density of water is $\rho g = 9800 N/m^3$.