**Problem Statements**

1. Solve each of the following for $x$. Always check for extraneous solutions.

   (a) $$ \frac{\ln(e^x)}{\ln(3)} = \frac{\ln(2)}{\ln(3)} $$
   
   $$ x = \frac{\ln(3)}{\ln(3)} $$

   (b) $$ 3^x = 8 $$

   Using the properties of logarithms,
   
   $$ \log(3^x) = \log(8) $$
   
   $$ x \cdot \log(3) = \frac{\log(8)}{\log(3)} $$

   $$ x = \frac{\log(8)}{\log(3)} $$

   (c) $$ \frac{15 \cdot e^{3x}}{100 + e^{2x}} = 3 $$

   $$ 15 = 3(100 + e^{2x}) $$

   $$ 15 = 300 + 3e^{2x} $$

   $$ -285 = 3e^{2x} $$

   $$ -95 = e^{2x} $$

   No solution.

2. Solve each of the following for $x$.

   (a) $$ (e^x + 9)(e^x - 2) = 0 $$

   $$ e^x + 9 = 0 $$

   $$ e^x = -9 $$

   $$ \ln(e^x) = \ln(-9) $$

   $$ x = \ln(2) $$

   (b) $$ \frac{\log_5(x+2)}{\log_5(6)} = \frac{\log_5(6)}{\log_5(6)} $$

   $$ \log_5((x+2)(x+8)) = \log_5(6) $$

   $$ \log_5((x+2)(x+8)) = \log_5(6) $$

   $$ \log_5(x^2 + 5x + 6) = \log_5(6) $$

   $$ x^2 + 5x + 6 = 6 $$

   $$ x^2 + 5x = 0 $$

   $$ x(x+5) = 0 $$

   $$ x = 0 $$

   Check:

   $$ \log_5(0) $$
Math 150 - Fall 2020
Math Learning Center

1. Solve the equation:
   \[ x^2 + 5x = 0 \]
   \[ x(x + 5) = 0 \]
   \[ x = 0 \quad \text{or} \quad x = -5 \]

2. Solve the inequality:
   \[ \log_2 (x + 4) = 5 \]
   \[ x + 4 = 2^5 \]
   \[ x = 32 - 4 \]
   \[ x = 28 \]

3. Solve the equation:
   \[ x^2 + 4x - 5 = 0 \]
   \[ (x + 5)(x - 1) = 0 \]
   \[ x = -5 \quad \text{or} \quad x = 1 \]

4. \[ \log_2 (x+2) = \log_2 4 \]
   \[ x + 2 = 4 \]
   \[ x = 2 \]

2. The number of bacteria in a culture after 1 day is given by the function \( y(t) = 100e^{0.05t} \).
   (a) What is the initial number of bacteria in the culture? 
   \[ y(0) = 100e^{0.05(0)} = 100 \cdot 1 = 100 \]
   (b) How many bacteria are there after 40 days? 
   \[ y(40) = 100e^{0.05(40)} = 100e^2 \approx 438 \]

(c) After how many days will there be 1000 bacteria? 

\[ 1000 = 100e^{0.05t} \]
\[ t = \frac{\ln(10)}{0.05} \approx 70 \text{ days} \]

6. If the half-life of an object is 5 years, how long would it take to go from 100 g to 25 g?

\[ \frac{100}{2^n} = 25 \]
\[ n = \frac{\ln(\frac{25}{100})}{\ln(2)} \approx 4 \text{ years} \]

7. It takes 4 hours for a substance to decay from 500 g to 100 g. What is the half-life of the substance?

\[ y = Pe^{-rt} \]
\[ \frac{100}{500} = e^{-r(4)} \]
\[ r = \frac{\ln(\frac{1}{5})}{4} \]

8. Antoine stands on a balcony and throws a ball to his dog, who is at ground level. The ball's height above the ground is given by \( h(t) = -16t^2 + 20t + 60 \). After 0.5 seconds, the ball reaches its maximum height. What is the height of the ball at this time? What is the maximum height of the ball?

\[ h(t) = -16t^2 + 20t + 60 \]
\[ h(0.5) = -16(0.5)^2 + 20(0.5) + 60 = 65.5 \text{ ft} \]
\[ k = \frac{b}{2a} = \frac{20}{2(-16)} = \frac{5}{8} \]

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4. Find the zeros and their multiplicities for each of the following functions. Then determine the end behavior of the graph.

(a) \( f(x) = -x^3 + 12x + 9 \)

\[ f(x) = -x^3 + 12x + 9 \]

End behavior: \( E.B. \quad \text{as } x \to \infty, f(x) \to -\infty \)

\( x = 3 \) is a zero with multiplicity 1.

(b) \( g(x) = 3x(x - 3) \)

\( g(x) = 3x(x - 3) \)

End behavior: \( E.B. \quad \text{as } x \to -\infty, g(x) \to -\infty \)

\( x = 3 \) is a zero with multiplicity 1.

5. Find the indicated information for the function \( f(x) = \frac{2x^2 - x - 3}{x^2 - 4x + 3} \).

Domain: \( \{x \in \mathbb{R} \mid x \neq 1, x \neq 3\} \)

Vertical Asymptote(s): \( x = 3 \)

x-intercept(s): \( (1, 0) \)

Horizontal Asymptote(s): \( y = \frac{2}{3} \)

6. Find \( f(g(x)) \) and \( g(f(x)) \) and their domain.

(a) \( f(x) = 2x + 1 \)

\[ f(g(x)) = f(2x + 1) = 2(2x + 1) + 1 = 4x + 3 \]

Domain of \( f(g(x)) = (\mathbb{R}, 0) \cup (0, \infty) \)

(b) \( g(x) = \frac{2}{x-1} \)

\[ g(f(x)) = g(2x + 1) = \frac{2}{2x + 1 - 1} = \frac{2}{2x} \]

Domain of \( g(f(x)) = (-\infty, 0) \cup (0, \infty) \)

7. Find the intervals where the inequalities are true.

(a) \( x^2 + 3x - 4 \geq 0 \)

\( x = -4 \) and \( x = 1 \)

(b) \( 3x^2 - x - 2 < 0 \)

\( x = \frac{2}{3} \) and \( x = -1 \)

\( [\frac{2}{3}, 1] \]

8. Find the inverse function and state its domain and range.

(a) \( y = x^2 - 2(x + 3) \)

\[ y = x^2 - 2(x + 3) \]

Inverse: \( D: (-\infty, 7] \)

Domain: \( (-\infty, 7] \)

\( y = 450 - 2x \) with area \( 11.25 \) square feet.
9. Describe the transformation(s) of the graph of \( f(x) = e^x \) that yields the graph of \( y = \frac{3e^x - 1}{3e^x} \).

Transformations:
- Vertical stretch
- Horizontal shrink \( \frac{1}{3} \)
- Reflection over x-axis

Domain: \((-\infty, \infty)\)

No x-intercept

No y-intercept

Horizontal Asymptote(s): \( y = 1 \)

No vertical asymptote(s)

10. Describe the transformation(s) of the graph of \( f(x) = \log_2(x) \) that yields the graph of \( y = -\log_2(0.5x) \).

Transformations:
- Horizontal shrink \( \frac{1}{2} \)
- Right \( \frac{1}{2} \)
- Reflect over x-axis

Domain: \((3, \infty)\)

No x-intercept

No y-intercept

Vertical Asymptote(s): \( x = \frac{5}{2} \)

\[ y = -\log_2(0.5x) = -\log_2(-6) \]

\[ \log_2(\log_2(4^w)) = \log_2(8^w \cdot \log_2(\frac{2}{3})) = \log_2(8^w) + \log_2(2) = \log_2(8^w) + 1 = \log_2(64) + 1 = 6 + 1 = 7 \]