Math 150 - Week-in-Review 8
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Problem Statements

1. Convert 75° to radians.

\[
\frac{75 \pi}{180} = \frac{75\pi}{180} = \frac{15\pi}{36} = \frac{5\pi}{12} \text{ radians}
\]

2. A circular sector created by a central angle of \(\frac{3}{5}\) radians has an area of 1080 ft², determine the radius of the circle.

\[
\frac{\text{Area of Sector}}{2} = \frac{r^2 \cdot \frac{3}{5}}{2} = \frac{1080}{2} = \frac{r^2}{\frac{3}{5}}
\]

\[
\frac{5}{3} (2160) = \left(r^2 \cdot \frac{3}{5}\right) \left(\frac{5}{3}\right)
\]

\[
(5)(720) = r^2
\]

\[
\frac{3600}{r^2} = 60
\]

\[
r = 60
\]

3. The planet Neptune has an orbit that is nearly circular. It orbits the Sun at a distance of 4497 million kilometers and completes one revolution every 165 years. How long is a full path of Neptune around the Sun? Then find the linear velocity of Neptune as it orbits the Sun.

\[
S = r \cdot \theta = 4497 \cdot 2\pi
\]

\[
\text{Linear Velocity} = \frac{S}{t} = \frac{8994\pi}{165} \text{ mill. km/yr}
\]

Full path: 8994π million km
4. Evaluate the six trigonometric functions for the following angles:

a) \( \sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \)

b) \( \cos \frac{4\pi}{3} = -\frac{1}{2} \)

c) \( \tan \frac{4\pi}{3} = -\sqrt{3} \)

d) \( \cot \frac{4\pi}{3} = -\sqrt{3} \)

e) \( \sec \frac{4\pi}{3} = -\frac{2}{1} \)

f) \( \csc \frac{4\pi}{3} = -\frac{2}{\sqrt{3}} \)

5. Find the exact value of the six trigonometric functions, given the following:

hypotenuse = 29, side opposite the angle = 21

\[
\sin \theta = \frac{21}{29}, \quad \csc \theta = \frac{29}{21},
\]

\[
\cos \theta = \frac{20}{29}, \quad \sec \theta = \frac{29}{20},
\]

\[
\tan \theta = \frac{21}{20}, \quad \cot \theta = \frac{20}{21}
\]

6. Given \( \sin \theta = \frac{4}{5} \) and \( \theta \) in Q1, use the trigonometric identities to find the exact value of each:

a. \( \cos(\theta) = \frac{\sqrt{33}}{5} \)

b. \( \cot(\theta) = \frac{\sqrt{33}}{4} \)

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

\[ \left(\frac{4}{5}\right)^2 + \cos^2 \theta = 1 \]

\[ \sin(90^\circ - \theta) = \cos(\theta) \]

\[ \cos(90^\circ - \theta) = \sin(\theta) \]
7. From a point on the ground 47 feet from the foot of a tree, the angle of elevation of the top of the tree is 30°. Find the height of the tree.

\[ \tan(30°) = \frac{y}{47/2} \]

\[ \frac{\sin(30°)}{\cos(30°)} = \frac{\frac{1}{2}}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{y}{47/2} \]

\[ y = \frac{47/2}{\sqrt{3}} \approx 27 \text{ ft.} \]
8. Find the exact value of \( x \) and \( y \).

\[
\sin(45^\circ) = \frac{x}{70} \quad \cos(45^\circ) = \frac{y}{70}
\]

\[
\frac{\sqrt{2}}{2} = \frac{x}{70} \quad \frac{\sqrt{2}}{2} = \frac{y}{70}
\]

\[
\frac{70\sqrt{2}}{2} = x \quad \frac{70\sqrt{2}}{2} = y
\]

\[x = 35\sqrt{2}, \quad y = 35\sqrt{2}\]

9. Let \((-24, 7)\) be a point on the terminal side of \( \theta \). Find the sine, cosine, and tangent of \( \theta \).

\[
7^2 + (-24)^2 = h^2
\]

\[49 + 576 = h^2 \quad 625 = h^2 \]

\[h = 25\]

\[
\sin\theta = \frac{7}{25} \quad \cos\theta = \frac{-24}{25} \quad \tan\theta = \frac{7}{-24}
\]

\[
\frac{24}{96} = \frac{1}{4}, \quad \frac{480}{576} = \frac{5}{6}
\]

10. Let \((3, -8)\) be a point on the terminal side of \( \theta \). Find the sine, cosine, and tangent of \( \theta \).

\[
3^2 + (-8)^2 = 9 + 64 = 73 = h^2
\]

\[
\sin\theta = \frac{-8}{\sqrt{73}} \\
\cos\theta = \frac{3}{\sqrt{73}} \\
\tan\theta = \frac{-8}{3}
\]

\[
\frac{1}{\tan\theta} = \frac{-5}{24}, \quad \frac{-5}{24} = -\frac{5}{24}
\]

\[
\text{Sec}\theta = \frac{7}{-2\sqrt{6}}
\]

11. Given \(\sin(\theta) = \frac{5}{7}\) and \(\tan(\theta) > 0\), find \(\tan(\theta)\) and \(\sec(\theta)\).

\[
x^2 + (-5)^2 = 7^2 \\
x^2 = 72 \\
x = -\sqrt{72} = -2\sqrt{18}
\]

\[
\sin(\pi - t) = \sin(-t + \pi) = \sin(t) \\
\sin(\pi + t) = \sin(t + \pi) = -\sin(t)
\]

\[\cos(\pi - t) = -\cos(t) \quad \cos(t) = \frac{5}{9}
\]

reflects over y-axis
\[ \cos(\pi + t) = -\cos(t) \]

\[ \tan(45^\circ) = \frac{23}{x} \]

\[ 1 = \frac{23}{x} \implies x = 23 \]

\[ \sin(45^\circ) = \frac{23}{r} \]

\[ r = \frac{23}{\sin(45^\circ)} = \frac{23}{\frac{\sqrt{2}}{2}} = 23 \cdot \frac{2}{\sqrt{2}} = \frac{46}{\sqrt{2}} \rightarrow 23\sqrt{2} \]

\[ \cot(\alpha) = \frac{6}{1} \] \[ \tan(\alpha) \csc(\alpha) \]

\[ \frac{\sin(\alpha)}{\cot(90^\circ - \alpha)} = \]

\[ \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \]

\[ \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \]

\[ \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \]

\[ \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \]

\[ \sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1 \]