Sections 12.1-12.4

Section 12.1

Problem 1. Find the center and radius of the sphere \( x^2 + y^2 + z^2 + 4x - 2y - 8z = 5 \). Does this sphere intersect the \( xz \) plane? If so, what is the intersection?

Problem 2. Find equation of the sphere with center \((1, 2, 5)\) that touches the \( xy \) plane.

Problem 3. Find the equation of the sphere if one of their diameters has endpoints \((5, 1, 5)\) and \((7, 3, 9)\).

Problem 4. What does \( y = 6 - x \) represent in \( \mathbb{R}^3 \)?

Problem 5. What does \( x^2 + z^2 = 16 \) represent in \( \mathbb{R}^3 \)?

Problem 6. Write a set of inequalities that describes the solid upper hemisphere \( x^2 + y^2 + z^2 = 9 \).

Section 12.2

Problem 7. Give a graphical interpretation of vector sum and vector difference.

Problem 8. Given \( \mathbf{a} = \langle -7, 1, 2 \rangle \) and \( \mathbf{b} = \langle 5, -1, 1 \rangle \), find a unit vector in the direction of \( \mathbf{a} + 2\mathbf{b} \).

Problem 9. For the picture seen below, write \( \mathbf{v} \) in terms of \( \mathbf{u} \) and \( \mathbf{w} \).
Section 12.3

Problem 10. Compute $a \cdot b$ if
a.) $a = \langle 4, 5, -1 \rangle$ and $b = \langle 2, 1, 3 \rangle$.
b.) $|a| = 2$, $|b| = 5$ and $\theta = 120^\circ$.
c.) $|a| = 6$, $|b| = 4$ and $a$ is perpendicular to $b$.
d.) $|a| = 6$, $|b| = 4$ and $a$ is parallel to $b$.

Problem 11. Are the vectors $-8i + 4j + 12k$ and $6i - 3j - 9k$ parallel, perpendicular, or neither?

Problem 12. The points $A(0, -1, 6)$, $B(2, 1, -3)$ and $C(5, 4, 2)$ form a triangle. Find $\angle C$.

Problem 13. Find the vector and scalar projection of $\langle 1, 2, 5 \rangle$ onto $\langle 0, 7, 4 \rangle$.

Section 12.4

Problem 14. Find the cross product of $\langle 1, 1, 3 \rangle$ and $\langle -2, -1, 5 \rangle$ and find the area of the parallelogram determined by the two vectors.

Problem 15. Find $|u \times v|$ and determine if $u \times v$ points in or out of the page.

Problem 16. Find two unit vectors that are orthogonal to the plane that passes through the points $P(1, 0, 1)$, $Q(2, 3, 4)$ and $R(2, 1, 1)$.

Problem 17. Determine whether each expression is meaningful or meaningless (circle one). If so, state whether the expression is a vector or a scalar.

a.) $a \cdot b$ meaningful (vector or scalar) meaningless
b.) $a \times b$ meaningful (vector or scalar) meaningless
c.) $a \cdot (b \times c)$ meaningful (vector or scalar) meaningless
d.) $a \times (b \times c)$ meaningful (vector or scalar) meaningless
e.) $(a \cdot b) \times (c \cdot d)$ meaningful (vector or scalar) meaningless
f.) $(a \times b) \cdot (c \times d)$ meaningful (vector or scalar) meaningless
g.) $a \times (b \cdot c)$ meaningful (vector or scalar) meaningless
h.) $|a|(b \times c)$ meaningful (vector or scalar) meaningless