1. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a
distance of 1 in. then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find
the position $u$ of the mass at any time $t$. Determine the frequency, period and amplitude of the motion.
2. A mass weighing 4 lb stretches a spring 1.5 in. The mass is given a positive displacement 2 in from its equilibrium position and released with no initial velocity. Assuming that there is no damping and the mass is acted on by an external force of $2 \cos 3t$ lb,

(a) Formulate the initial value problem describing the motion of mass

(b) Solve the initial value problem.

(c) If the given external force is replaced by a force $4 \cos \omega t$ of frequency $\omega$, find the value of $\omega$ for which resonance occurs.
3. A 3 kg object is attached to spring and will stretch the spring 392 mm by itself. There is no damping in the system and a forcing function of the form \( F(t) = 10 \cos(\omega t) \) is attached to the object and the system will experience resonance. If the object is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward find the displacement at any time \( t \).
4. A spring is stretched 6 in by a mass that weighs 8 lb. The mass is attached to a dashpot mechanism that has a damping constant of 0.25 lb·s/ft and is acted by an external force of \(4 \cos 2t\) lb.

(a) Find the steady-state response of this system.

(b) If the given mass is replaced by a mass \(m\), determine the value of \(m\) for which the amplitude of the steady-state response is maximum.
5. Use the definition to find the Laplace transforms of
(a) \( f(t) = e^{at} \) where \( a \) is a non zero real number.

(b) \( f(t) = \begin{cases} 5 - t & 0 \leq t < 2 \\ 3t & 2 \leq t. \end{cases} \)
6. Find the inverse Laplace transform of the following functions.

(a) \( F(s) = \frac{4}{(s - 2)^5} \)

(b) \( F(s) = \frac{8s^2 - 4s + 12}{s(s^2 + 4)} \)

(c) \( F(s) = \frac{2s - 3}{s^2 + 2s + 10} \)
7. Use the Laplace transform to solve the given initial value problem

(a) \( y'' + 3y' + 2y = 4t, \quad y(0) = 1, \quad y'(0) = 0. \)
(b) $y'' + 9y = \cos 2t, \quad y(0) = 0, \quad y'(0) = 1$. 