1. Determine the radius of convergence for the following power series:

(a) \[ \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x + 2)^n}{3^n} \]

2. Determine a lower bound for the radius of convergence of series solutions of the differential equation

\[ (x^3 - x^2 + 2x - 2) y'' + (2x + 1)y' + 4x^2 y = 0 \]

about the point \( x = 0 \) and about the point \( x = -\frac{1}{3} \).
3. Write the power series expansion of the following series

\[ f(x) = x \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - (x+1)^2 \sum_{n=0}^{\infty} a_n x^n. \]

Then find \( f^{(10)}(0) \) assuming that the series has a positive radius of convergence about \( x_0 = 0 \).
4. For the Airy’s equation $y'' - xy = 0$

(a) Find a lower bound for the radius of convergence of its power series solution about $x_0 = 0$.
(b) Seek its power series solution about $x_0 = 0$; find the recurrence relation.
(c) Find the general term of each solution $y_1(x)$ and $y_2(x)$, and find their radii of convergence.
(d) Find the first four terms in each of two solutions $y_1$ and $y_2$. Show that $W[y_1, y_2](0) \neq 0$. 
5. For the equation \((x^2 + 2x + 2)y'' + (2x + 2)y' - 2y = 0\)

(a) Find a lower bound for the radius of convergence of its power series solution about \(x_0 = 1\).
(b) Seek its power series solution about \(x_0 = 1\); find the recurrence relation.
(c) Find the general term of each solution \(y_1(x)\) and \(y_2(x)\), and find their radii of convergence.
(d) Find the first four terms in each of two solutions \(y_1\) and \(y_2\). Show that \(W[y_1, y_2](0) \neq 0\).
(e) Investigate if the solutions can be recognized as elementary functions of calculus.
6. Solve the equation $y'' + e^x y = 0$ using the power series method. Only find the terms up to $x^5$. 
7. For the following equation, determine $\phi''(0)$ and $\phi'''(0)$ if $y = \phi(x)$ is a solution of the given initial-value problem.

$$y'' + x^2 y' + (\sin x)y = 0; \quad y(0) = a_0, \quad y'(0) = a_1.$$ 

Then express the solution $\phi(x)$ as a power series centered at 0. Only write the terms up to $x^3$. 