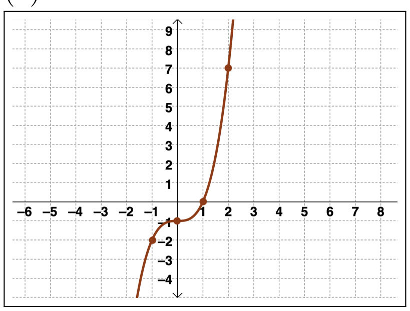
WIR Solutions: Sections 5.7 and 5.8

This document contains the answers to the posed problems. Video solutions will be added as they are produced.

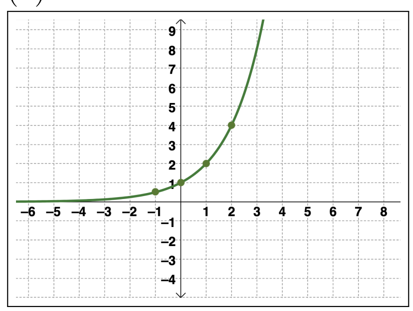
Section 5.7

(1) For each graph given below, state the parent function and then identify the transformations of the parent function and write the resulting function.

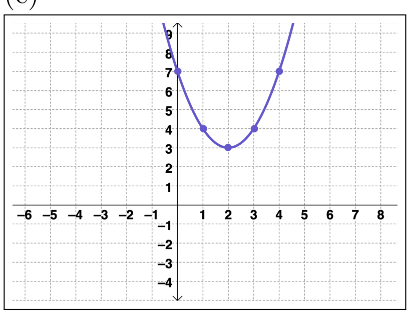
(a)



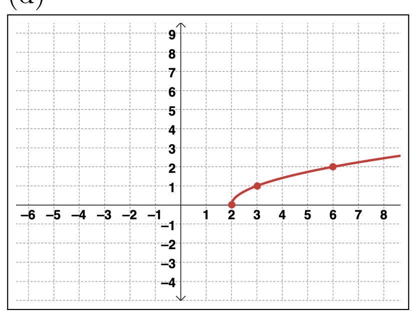
(b)



(c)



(d)



Answers:

(a) The parent function is $f(x) = x^3$. In the graph, $f(x)$ is shifted down one unit. The resulting function is $f_1(x) = x^3 + 1$.

(b) The parent function is $g(x) = 2^x$. In the graph $g(x)$ is not shifted.

(c) The parent function is $h(x) = x^2$. In the graph $h(x)$ is shifted right two units and up three units. The resulting function is $h_1(x) = (x-2)^2 + 3$.

(d) The parent function is $p(x) = \sqrt{x}$. In the graph $p(x$ is shifted right two units. The resulting function is $p_1(x) = \sqrt{x-2}$.

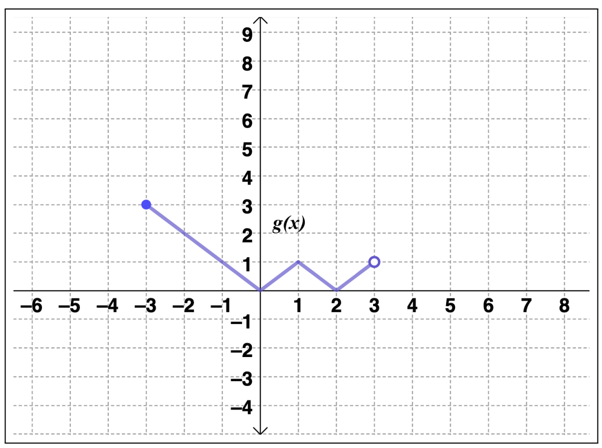
(2) State the parent function for $g(x) = -2 \cdot e^{x+3}$ and then the list of transformations (in the correct order) needed to graph the function.

Answer: The parent function for $g(x)$ is $g_p(x) = e^x$. The correct order of transformations of $g_p(x) = e^x$ to obtain $g(x)$ is: (1) shift left three units, (2) vertically expanded by a factor of 2, and then (3) reflect across the x-axis.

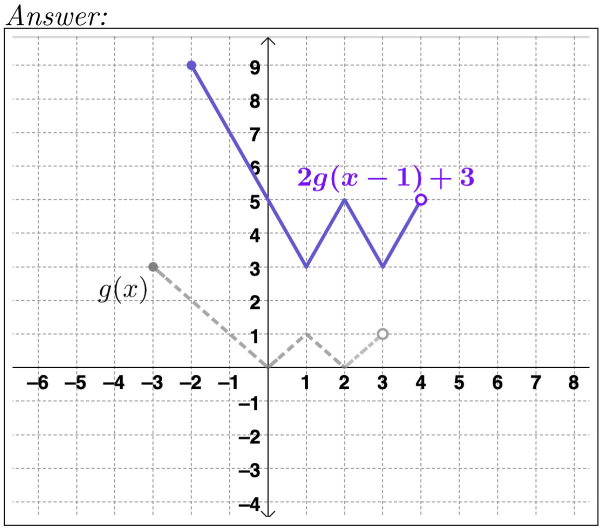
(3) Write the equation of the function, $g(x)$, whose graph is the result of $f(x) = \sqrt{x}$ undergoing the transformation of shifted left 3 units, vertically compressed by a factor of 3, and then shifted down 8 units.

Answer: $g(x) = \frac{1}{3}\sqrt{x+3} - 8$

(4) Given the graph of $g(x)$ below, draw the graph of $2g(x-1)+3$.



Answer:



(5) Given $g(x) = -x^2 - 1$, $h(x) = |x+1|$ and the graph of $f(x)$ below, find

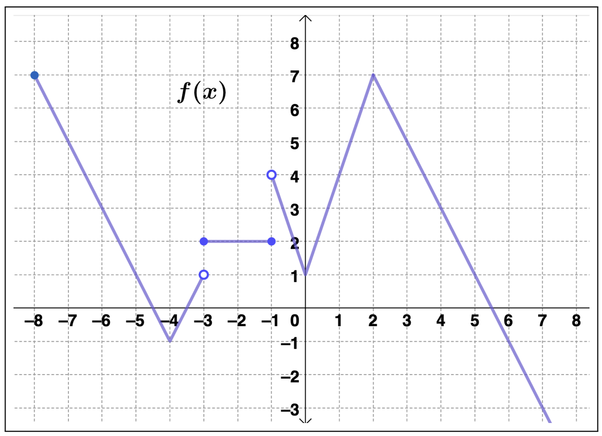
(a) $f(f(-5))$
Answer: 4

(b) $(g \circ f)(3)$
Answer: -26

(c) $(f \circ g)(0)$
Answer: 2

(d) $g(h(2))$
Answer: -10

(e) $(g \circ f \circ h)(-3)$
Answer: -50



Section 5.8

(6) Without using a calculator, evaluate $7^{2\log_7(4)}$.
Answer: 16

(7) Express $\frac{1}{2}\log_4(x+1) + \log_4(x) - 4\log_4(2x^2-1)$ as a single logarithm. When necessary, assume all variables represent positive real numbers.
Answer: $\log_4\left(\frac{x(x+1)^{1/2}}{(2x^2-1)^4}\right)$

(8) Use the properties of logarithms to fully expand and simplify the expression $\ln\left(\sqrt[3]{\frac{2x^3}{e^2y^3}}\right)$. When necessary, assume all variables represent positive real numbers.
Answer: $\frac{1}{3}\ln(2) + \ln(x) - \frac{2}{3} - \ln(y)$

(9) Use properties of logarithms to determine whether each statement below is true or false.

(a) $\log_4\left(\frac{16}{x}\right) = 2 - \log_4(x)$
Answer: True

(b) $\frac{\ln(5x)}{\ln(10x)} = \ln\left(\frac{1}{2}\right)$
Answer: False

(c) $\log(10)^{2x} = xe^{\ln(2)}$
Answer: True

(10) For each of the logarithmic functions below, state the (a) domain, (b) range, (c) end behaviors (i.e., behavior of the function values as $x \to \pm\infty$), (d) x-intercept(s), and (e) y-intercept.

(a) $f(x) = \log_{\frac{1}{2}}(x)$
Answer:
(a) The domain of $f(x)$ is $(0, \infty)$.
(b) The range is of $f(x)$ is $(-\infty, \infty)$.
(c) As $x \to 0^+, f(x) \to \infty$. As $x \to \infty, f(x) \to -\infty$.
(d) The x-intercept is $(1, 0)$
(e) No y-intercept.

(b) $g(x) = \ln(x)$
Answer:
(a) The domain of $g(x)$ is $(0, \infty)$.
(b) The range is of $g(x)$ is $(-\infty, \infty)$.
(c) As $x \to 0^+, g(x) \to -\infty$. As $x \to \infty, g(x) \to \infty$.
(d) The x-intercept is $(1, 0)$
(e) No y-intercept.

(11) State the domain of each algebraic function, using interval notation.

(a) $h(x) = \frac{\ln(x+2)}{e^{\sqrt{3x-1}}}$
Answer: $\left[\frac{1}{3}, \infty\right)$

(b) $g(x) = \ln(4x - 8) + \ln(x + 1)$
Answer: $(2, \infty)$

(c) $f(x) = \frac{2^x}{\ln(x) - 1}$
Answer: $(0, e) \cup (e, \infty)$

(12) Solve each of the following for x. Leave your answers in exact form.

(a) $3^{x-4} = 5$

Answer: $x = \log_3(5) + 4 = \dfrac{\ln(5)}{\ln(3)} + 4$

(b) $2e^{2x} - e^{x} = 3$

Answer: $x = \ln\left(\dfrac{3}{2}\right)$

(c) $\log_3(4 - 2x) + \ln(e)^2 = \log_3(x - 1)$

Answer: $x = \dfrac{37}{19}$

(d) $\log_5(8 - x) = \log_5(48) - \log_5(-x)$

Answer: $x = -4$