WIR: EXAM 1 REVIEW

(1) A line \( L \) passes through the points \((7, 2)\) and \((2, 5)\). If \( x \) increases by 4, what is the change in \( y \)?

(2) Laura is planning to buy one 5-lb bag of sugar, two 5-lb bags of flour, four 1-gal cartons of milk, and one 1-dozen carton of large eggs. The prices of these items in three neighborhood supermarkets are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Sugar 5-lb bag</th>
<th>Flour 5-lb bag</th>
<th>Milk 1-gal carton</th>
<th>Eggs 1-dozen carton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supermarket 1</td>
<td>$3.29</td>
<td>$3.29</td>
<td>$2.79</td>
<td>$2.99</td>
</tr>
<tr>
<td>Supermarket 2</td>
<td>$2.59</td>
<td>$2.99</td>
<td>$2.89</td>
<td>$3.49</td>
</tr>
<tr>
<td>Supermarket 3</td>
<td>$3.19</td>
<td>$3.69</td>
<td>$3.59</td>
<td>$3.99</td>
</tr>
</tbody>
</table>

(a) Write a \( 4 \times 3 \) matrix \( A \) to represent the prices of items in the three supermarkets.
(b) Write a row matrix \( B \) to represent the quantities of the items that Laura plans to purchase in the three supermarkets.
(c) Use matrix multiplication to find a matrix \( C \) that represents Laura’s total cost at each supermarket. Which supermarket should she buy from to minimize her cost? (Assuming she only shops at one supermarket)

(3) Melanie bought a brand-new car in 2008. In 2013 the car was worth $14,500 and in 2028 she sold it to the salvage yard (or scrap yard) for $5,500. Assuming a linear depreciation model, find an equation that gives the value, \( V \), of the car \( t \) years after its initial purchase. Use the equation to determine the original amount that Melanie paid for the car in 2008.

(4) A company makes and sells skateboards. The total profit (in dollars) of producing and selling \( x \) skateboards is \( P(x) = 110x - 17,050 \). The company sells the skateboards for $200 each. Find the company’s break-even point and interpret the meaning of this point.

(5) Determine which of the following matrices below are in row reduced echelon form (RREF).

\[
\begin{pmatrix}
1 & 0 & -3 & 0 \\
0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 0 & 1 & 6 \\
0 & 1 & 0 & 4 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad
\begin{pmatrix}
1 & 0 & 2 & 3 \\
0 & 1 & 0 & 7 \\
0 & 0 & 1 & 9
\end{pmatrix}
\]

(a) 
(b) 
(c)

(6) Given the matrix equation below, find matrix \( D \).

\[
2 \begin{bmatrix} 3 & 1 & -1 \\ 0 & a & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 & x \\ 6 & 2 & -1 \end{bmatrix}^T + \begin{bmatrix} 1 & -y \\ 3 & 6 \end{bmatrix} = D
\]

(7) Given matrix \( A \) has dimensions \( 3 \times 4 \) and matrix \( B \) has dimensions \( 2 \times 3 \). Determine if \( B A^T \) exists. If it does, give the dimensions of the matrix \( B A^T \).

(8) Given the dimensions of matrix \( E \) are \( 3 \times 4 \), find the dimensions of matrix \( F \) for which \( 4E + F \) is defined?
(9) Write the correct augmented matrix for the system of linear equations given below.

\[
\begin{align*}
-3y + z &= -7 \\
y &= \frac{1}{2}x + 4 \\
2x - 2y + 5z &= -2
\end{align*}
\]

(10) The parameterized solution to a system of linear equations with infinitely many solutions is given by \((w, x, y, z) = (3 - 2t, 4, t, 1 - t)\) where \(t\) is any real number. Determine if the values below are a particular solution to the given system.

(a) \((w, x, y, z) = (3, 4, 0, 1)\)  
(b) \((w, x, y, z) = (1, 4, 1, 1)\)  
(c) \((w, x, y, z) = (5, 4, -1, 2)\)

(11) Given a matrix below, which row operation must be performed to pivot about the entry in row one column one?

\[
\begin{bmatrix}
1 & -2 & -1 & 3 \\
0 & 6 & 0 & 1 \\
-2 & 0 & 3 & 2
\end{bmatrix}
\longrightarrow
\begin{bmatrix}
1 & -2 & -1 & 3 \\
0 & 6 & 0 & 1 \\
0 & -4 & 1 & 8
\end{bmatrix}
\]

(12) Without the aid of technology, find the solution to the system of linear equations below.

\[
\begin{align*}
-3x - 6y &= -12 \\
-2x + 3y &= 15
\end{align*}
\]

(13) Without the aid of technology, find the solution to the system of linear equations given by \(y_1\) (graphed below) and \(-6x + 4y = 0\).

(14) Use technology to find the solution to the following systems of linear equations.

(a) \[
\begin{align*}
2x + y - z &= 1 \\
3x + 4y + 2z &= 13 \\
x - 5y - 2z &= 0
\end{align*}
\]

(b) \[
\begin{align*}
x + y + 2z &= -2 \\
3x - y + 14z &= 6 \\
x + 2y &= -5
\end{align*}
\]

(15) Given matrices \(A\) and \(B\) below, find the products \(AB\) and \(BA\), if they exist.

\[
A = \begin{bmatrix} x & 0 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -3 & -2 \\ 1 & 4 & 3 \end{bmatrix}
\]
(16) Each day you feed your dog a mixture of three kinds of food: Kibble, Bits, and Chunks. Matrix $M$ shows the amount of vitamins $A$, $B$, and $C$ (in milligrams per cup) for each type of food. Matrix $N$ shows the number of cups of each type of food consumed by the dog each day.

$$M = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 4 & 5 \\ 2 & 5 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1.2 \\ .8 \\ .5 \end{bmatrix}$$

Given $A = MN$, calculate the entry $a_{21}$ in matrix $A$ and give the correct interpretation for $a_{21}$.

(17) The quantity demanded for a certain brand of portable CD players is 200 units when the unit price is set at $72. The quantity demanded increases by 1000 units when the unit price decreases by $40. The corresponding supply equation is $p(x) = .06x + 10$ where $p(x)$ is the price in dollars at which $x$ CD players will be supplied.

(a) Find the demand equation, assuming the demand equation is linear.
(b) Find the equilibrium point and then interpret the meaning of the equilibrium point.
(c) If the price is $60, will there be a surplus or shortage of CD players. Explain your reasoning.
(d) How many CD players will be demanded if they are given away for free?
(e) Suppliers will only provide the items if the price is above what value?

(18) Given a matrix $A$ below, use row operations to get the matrix in reduced row echelon form.

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -1 \\ 2 & -3 & 5 \end{bmatrix}$$

(19) When 150 items are produced and sold, a company has total costs of $27,000 and the cost per item is 85. The total profit from selling 150 units is $45,000.

(a) Find the cost equation $C(x)$.
(b) Find the revenue equation $R(x)$.
(c) Find the profit equation $P(x)$.

(20) Write a system of equations that represents the following problem. Do not solve the system.
An executive of Trident Communications recently traveled to London, Paris, and Rome. He paid $215, $260, and $250 per night for lodging in London, Paris and Rome, respectively, and his hotel bills totaled $3420. He spend $100, $125, and $110 per day for his meals in London, Paris, and Rome, respectively, and his expenses for meals totaled $2000. If he spent twice as many days in London as he did in Paris and Rome combined, how many days did he stay in each city?