WIR Solutions: Sections 3-4 and 4-1

This document contains the answers and video solutions to the posed problems. Click the red box to watch the video solution. You can also watch all videos by viewing the Section 3.4 and 4.1 playlist. Closed captions are available for all videos and the speed of the videos may be adjusted inside of "Settings" or the cog in the bottom right corner.

Section 3.4

(1) Write the initial simplex tableau, if possible, for the linear programming problem. If it is not possible to write the initial simplex tableau, explain why not.
Objective: Maximize $P = 15x + 12y$ subject to:

$$\begin{cases} 5 \geq 2x + y \\ 3x + 6y \leq 25 \\ 3x \geq 4y \\ x, y \geq 0 \end{cases}$$

Answer: This is a standard maximization problem and the initial tableau is:

$$\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & P & \text{constant} \\ 2 & 1 & 1 & 0 & 0 & 0 & 5 \\ 3 & 6 & 0 & 1 & 0 & 0 & 25 \\ -3 & 4 & 0 & 0 & 1 & 0 & 0 \\ \hline -15 & -12 & 0 & 0 & 0 & 1 & 0 \end{array}$$

Click here for video solution to #1.

(2) Write the linear programming problem which corresponds to the following initial simplex tableau.

$$\begin{array}{cccccc|c} x & y & z & s_1 & s_2 & P & \text{const.} \\ 2 & 1 & 0 & 1 & 0 & 0 & 10 \\ 2 & 3 & 4 & 0 & 1 & 0 & 18 \\ \hline -4 & -3 & -1 & 0 & 0 & 1 & 0 \end{array}$$

Answer: Maximize $P = 4x + 3y + z$ subject to

$$\begin{cases} 2x + y \leq 10 \\ 2x + 3y + 4z \leq 18 \\ x, y \geq 0 \end{cases}$$

Click here for video solution to #2.

(3) For the tableau given below,

(a) State the basic and non-basic variables.

(b) Give the value of each variable.

(c) Determine if the given tableau is a final tableau. If it is not the final tableau, give the next pivot element.

$$\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \\ -4 & 0 & -1/2 & 1 & 0 & -3/2 & 0 & 300 \\ 1 & 0 & 1/2 & 0 & 1 & -1/2 & 0 & 150 \\ 2 & 1 & 1/2 & 0 & 0 & 1/2 & 0 & 200 \\ \hline 12 & 0 & -2 & 0 & 0 & 4 & 1 & 1600 \end{array}$$

Answer:

(a) Basic variables are y, s_1, s_2, and P.
The non-basic variables are x, z, and s_3.

(b) $x = 0, y = 200, z = 0, s_1 = 300, s_2 = 150, s_3 = 0, P = 1600$

(c) The tableau is not in final form. The pivot element is in column 3, row 2.

Click here for video solution to #3.

(4) Use the constraints given below and the optimal solution of $(x, y) = (0, 30)$ to state the value of each slack variable.

$$\begin{cases} 6x + 5y \leq 500 \\ 20x + 30y \leq 900 \\ x, y \geq 0 \end{cases}$$

Answer: $s_1 = 350, s_2 = 0$

Click here for video solution to #4.

(5) Use the simplex method to solve the linear programming problem given below. For each stage of the simplex process, give the value of each variable for each tableau.
Use the online website to help with pivoting on the proper elements in the tableaus.
Maximize $P = 4x + 8y + 6z$ subject to:

$$\begin{cases} 2x + 3y + z \leq 900 \\ 3x + y \leq 350 - z \\ 4x + z - 400 \leq -2y \\ x, y, z \geq 0 \end{cases}$$

Answer: Initial tableau is below. First pivot element is column 2, row 3. The corner point corresponding to this tableau is $x = 0, y = 0, z = 0$. The values of the other variables are $s_1 = 9000, s_2 = 350, s_3 = 400$, and $P = 0$.

$$\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \\ 2 & 3 & 1 & 1 & 0 & 0 & 0 & 900 \\ 3 & 1 & 1 & 0 & 1 & 0 & 0 & 350 \\ 4 & 2 & 1 & 0 & 0 & 1 & 0 & 400 \\ \hline -4 & -8 & -6 & 0 & 0 & 0 & 1 & 0 \end{array}$$

Tableau 2 is below. The tableau is not in final form. The second pivot element is in column 3, row 2. The corner point correspoinding to this tableau is $x = 0, y = 200, z = 0$. The values of the other variables are $s_1 = 300, s_2 = 150, s_3 = 0$, and $P = 1600$.

$$\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \\ -4 & 0 & -1/2 & 1 & 0 & -3/2 & 0 & 300 \\ 1 & 0 & 1/2 & 0 & 1 & -1/2 & 0 & 150 \\ 2 & 1 & 1/2 & 0 & 0 & 1/2 & 0 & 200 \\ \hline 12 & 0 & -2 & 0 & 0 & 4 & 1 & 1600 \end{array}$$

Tableau 3 is below. The tableau is in final form. The final solution is $x = 0, y = 50, z = 300, s_1 = 450, s_2 = 0, s_3 = 0, P = 2200$. Thus, P attains a maximum value of 2200 at $(x, y, z) = (0, 50, 300)$. There will be 450 units leftover for the inequality $2x + 3y + x \leq 900$

$$\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \\ -3 & 0 & 0 & 1 & 1 & -2 & 0 & 450 \\ 2 & 0 & 1 & 0 & 2 & -1 & 0 & 300 \\ 1 & 1 & 0 & 0 & -1 & 1 & 0 & 50 \\ \hline 16 & 0 & 0 & 0 & 4 & 2 & 1 & 2200 \end{array}$$

Click here for video solution to #5.

(6) **Manufacturing Problem:** A factory manufactures chairs, tables and bookcases each requiring the use of three operations: Cutting, Assembly, and Finishing. The cutting can be used at most 600 hours; assembly at most 500 hours; and finishing at most 300 hours. A chair requires 1 hour of cutting, 1 hour of assembly, and 1 hour of finishing; a table needs 1 hour of cutting, 2 hours of assembly, and 1 hour of finishing; and a bookcase requires 3 hours of cutting, 1 hour of assembly, and 1 hour of finishing. If the profit is \$20 per unit for a chair, \$30 for a table, and \$25 for a bookcase, how many units of each should be manufactured to maximize profit?

For the manufacturing problem given above, we want to maximize $P = 20x+30y+25z$ subject to:

$$\begin{cases} x + y + 3z \leq 600 \\ x + 2y + z \leq 500 \\ x + y + z \leq 300 \\ x, y, z \geq 0 \end{cases}$$

where x is the number of chairs manufactured; y is the number of tables manufactured; z is the number of bookcases manufactured; and P is the profit (in dollars) obtained by the company. *Note: It is good practice to make sure you can set up the objective function and constraints for this linear programming problem as well as the initial simplex tableau. It is also recommended you perform simplex to obtain the final tableau given below.*

After using the simplex method to solve this problem the final tableau is given below. Interpret the tableau below in the context of the problem.

$$\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \\ \end{array}$$
$$\left[\begin{array}{ccccccc|c} -2 & 0 & 0 & 1 & 2 & -5 & 0 & 100 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 200 \\ 1 & 0 & 1 & 0 & -1 & 2 & 0 & 100 \\ \hline 5 & 0 & 0 & 0 & 5 & 20 & 1 & 8500 \end{array}\right]$$

Answer: The company will obtain a maximum profit of \$8500 when they manufacture no chairs, 200 tables, and 100 bookcases. They will use all the assembly and finishing hours but will have 100 cutting hours leftover.

Click here for video solution to #6.

Section 4.1

(7) In an experiment the numbers −1, −2, 0, and 2 are written on equal sized pieces of paper and placed in a box. Two pieces of paper are drawn at the same time and the sum is noted. State the sample space.

Answer: $S = \{-3, -2, -1, 0, 1, 2\}$

Click here for video solution to #7

(8) An experiment consists of rolling two distinguishable four-sided die and noting the sum of the numbers that land uppermost.

(a) State the sample space.
(b) List of outcomes in the event, E, a three is rolled.
(c) Give one example of a simple event for this experiment.

Answer:
(a) $S = \{2, 3, 4, 5, 6, 7, 8\}$
(b) $E = \{4, 5, 6, 7\}$
(c) G is the event double fours are rolled. $G = \{8\}$

Click here for video solution #8.

(9) Let $S = \{h, o, l, i, d, a, y\}$ with events $E = \{d, a, y\}$, $F = \{l, i, d\}$, $G = \{y\}$ and $H = \{o, i, l\}$. Use set notation to describe the given event.

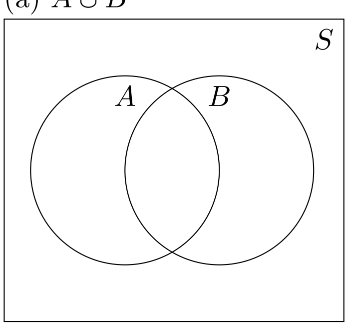
(a) $(F \cap G)^C$
(b) $(F \cup G)^C$
(c) $G^C \cap (E \cup F)$

Answer:
(a) S
(b)) $\{h, o, a\}$
(c) $\{d\}$

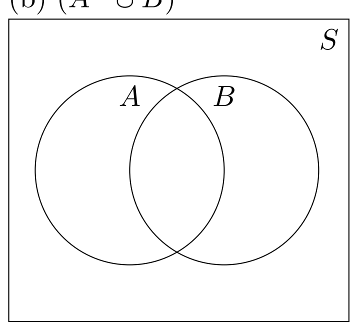
Click here for video solution #9.

(10) Shade the region(s) of the two-circle Venn diagram corresponding to the events:

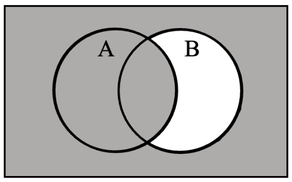
(a) $A \cup B^C$

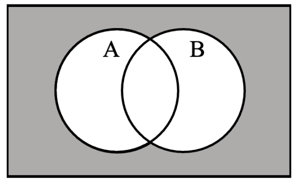


(b) $(A^C \cup B)^C$



Answer:





Click here for video solution #10.

(11) An experiment consists of drawing a card from a standard deck of 52 cards, noting the color, and then rolling one standard six-sided die, noting the number rolled. Let

- E := the event "a black card is drawn"
- F := the event " a number less than 4 is rolled"
- G := the event "an odd number is rolled"

(a) Describe and list the outcomes in the event $E \cap G$.
(b) Describe and list the outcomes in the event $F^C \cup G$.
(c) Write the symbolic notation for the event "a red card is drawn or an even number less than 4 is rolled."

Answer:

(a) A black card is drawn and an odd number is rolled. $E \cap G = \{(black, 1), (black, 3), (black, 5)$
(b) A number greater than or equal to 4 is rolled or an odd number is rolled. $F^C \cup G = \{(black, 4), (red, 4), (black, 5), (red, 5), (black, 6), (red, 6), (black, 1), (red, 1), (black, 3), (red, 3)\}$
(c) $E^C \cup (G^C \cap F)$

Click here for video solution to #11.