WIR Solutions: Exam 2 Review

This document contains the answers and video solutions to the posed problems. Click the red box to watch the video solution. You can also watch all videos by viewing the Exam 2 Review playlist. Closed captions are available for all videos and the speed of the videos may be adjusted inside of "Settings" or the cog in the bottom right corner.

(1) An experiment consists of rolling a fair, four-sided die and observing the number that lands uppermost. If the result is an even number, we draw a card from a standard well-shuffled 52 card deck and record the color. If the result is odd, we flip a fair coin and record whether the side that lands uppermost is heads or tails.
Let $H =$ heads, $T =$tails, $R =$ Red, and $B =$ Black

(a) Find an appropriate sample space for this experiment.
Answer: $S = \{(1, H), (1, T), (2, R), (2, B), (3, H), (3, T), (4, R), (4, B)\}$

(b) Write (or list) the events:

- E is the event a red card is drawn.
Answer: $E = \{(2, R), (4, R)\}$
- F is the event a tails occurs.
Answer: $F = \{(1, T), (3, T)\}$
- G is the event an even number is rolled.
Answer: $G = \{(2, R), (2, B), (4, R), (4, B)\}$

(c) List and describe the event $E \cup F^C \cap G$.
Answer: $E \cup F^C \cap G = G$

Click here for video solution to #1.

(2) Before Thanksgiving a group of Aggies were surveyed and asked to choose their favorite side dish at Thanksgiving dinner. The survey results are displayed in the table below.

	Candied Yams	Corn Bread Dressing	Roasted Brussel Sprouts	Green Beans	Other	Total
Freshman	5	20	30	10	20	85
Sophomore	7	14	17	11	6	55
Junior	2	32	25	20	10	89
Senior	7	19	30	7	8	71
Total	21	85	102	48	44	300

If one Aggie is randomly selected from this group, what is the probability that the Aggie:

(a) likes roasted brussel sprouts?

Answer: $\frac{102}{300}$

(b) is a freshman?

Answer: $\frac{85}{300}$

(c) is an sophomore whose favorite side dish was green beans?

Answer: $\frac{11}{300}$

(d) is a senior or junior or whose favorite side dish was candied yams?

Answer: $\frac{89+71+5+7}{300} = \frac{172}{300}$

Click here for video solution to #2.

(3) Let E and F be two events of an experiment with sample space S. Suppose that $P(E) = 0.4, P(F) = 0.3$, and $P(E \cup F) = 0.6$. Find $P(E^C \cap F)$.

Answer: $P(E^C \cap F) = 0.2$

Click here for video solution to #3.

(4) An experiment has a sample space of $S = \{i,\ l,\ o,\ v,\ e,\ m,\ a,\ t,\ h\}$, with events $E = \{t, a, i, l\}$, $F = \{l,\ o,\ v,\ e\}$, and $G = \{m,\ a,\ t,\ h\}$. Find each of the following.

(a) $E \cup F^C$

Answer: $E \cup F^C = \{t, a, i, l, m, h\}$

(b) $E \cap F^C$

Answer: $E \cap F^C = \{t, a, i\}$

(c) $(F \cap G)^C$

Answer: $(F \cap G)^C = S$

Click here for video solution to #4.

(5) Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ be the sample space associated with the following ***partial*** probability distribution table.

Outcome	s_1	s_2	s_3	s_4	s_5
Probability	$\frac{3}{20}$	$\frac{7}{20}$		$\frac{4}{20}$	$\frac{1}{20}$

Let $E = \{s_3, s_4\}, F = \{s_4, s_5\}$, and $G = \{s_1, s_2, s_3\}$, find:

(a) $P(E)$.

Answer: $\frac{9}{20}$

(b) $P(F^C)$.

Answer: $\frac{15}{20}$

(c) $P(E \cap G)$.

Answer: $\frac{5}{20}$

(d) $P(G \cup E)$.

Answer: $\frac{19}{20}$

(e) $P(F \cap G)$.

Answer: 0

Click here for video solution to #5.

(6) An experiment consists of rolling two distinguishable four-sided die and observing the two numbers that land uppermost.

(a) Let Y be the random variable denoting the number of fours that land uppermost on the dice. Create the probability distribution for Y.

Answer:

Y	0	1	2
$P(Y)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

(b) Let X be the random variable denoting the product of the two numbers. Create the probability distribution for X and then find the expected value for X.

Answer:

X	1	2	3	4	6	8	9	12	16
$P(X)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

$E(X) = 6.25$

Click here for video solution to #6.

(7) Use the given ***partial*** histogram below, to answer the questions that follow.

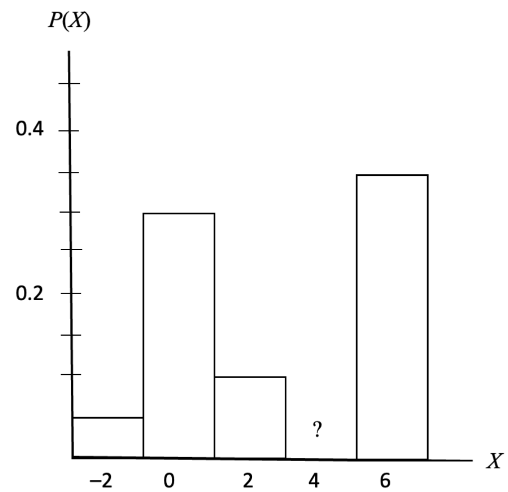
(a) Compute the value of $P(X = 4)$.
Answer: $P(X = 4) = 0.2$

(b) Compute the value of $P(X \leq 2)$.
Answer: $P(X \leq 2) = 0.45$

(c) Compute the value of $P(-2 < X \leq 4)$.
Answer: $P(-2 < X \leq 4) = 0.3 + 0.1 + 0.2 = 0.6$

(d) Compute the value of $P(X \geq 0)$.
Answer: $P(X \geq 0) = 1 - 0.05 = 0.95$

(e) Calculate $E(X)$.
Answer: $E(X) = -2(0.05) + 0(0.3) + 2(0.1) + 4(0.2) + 6(0.35) = 3$



Click here for video solution to #7.

(8) You pay \$8 to roll two fair five-sided dice (each numbered 1-5), noting the numbers rolled on each die. If you roll a double, you win \$20. If you roll different numbers that have a sum less than 4, you win \$40. Otherwise you win nothing.

(a) Create the probability distribution table for the random variable X where X represents your net winnings.
Answer:

X	12	32	-8
$P(X)$	$\frac{5}{25}$	$\frac{2}{25}$	$\frac{18}{25}$

(b) Calculate $E(X)$ and interpret your answer. Round your answer to two decimal places.
Answer: $E(X) = 12(\frac{5}{25}) + 32(\frac{2}{25}) + (-8)(\frac{18}{25}) = -0.8$
You can expect to lose \$0.80 by playing this game.

(9) **For the linear programming problem below, define the variables, set up the objective function and constraints, then determine if it is a standard maximization problem.** A company manufactures Products A, B, and C. Each product is processed in three departments: Dept. I, Dept. II, and Dept. III. The total available labor-hours, per week, for departments I, II, and III are 800 hours, 1080 hours, and 840 hours, respectively. The number of labor-hours required to manufacture each product in each department, as well as the profit realized per product for each of the three products are displayed in the table below.

	Product A	Product B	Product C
Dept. I	3 hours	1 hour	3 hours
Dept. II	4 hours	2 hours	2 hours
Dept. III	2 hours	3 hours	1 hour
Profit	\$16	\$20	\$15

How many units of each product should the company produce each week to maximize their profit?

Answer:

(a) Let x by the number of Product A manufactured each week.
Let y by the number of Product B manufactured each week.
Let z by the number of Product C manufactured each week.

(b) Objective function: $P = 16x + 20y + 15z$
Constraints:

$$\begin{cases} 3x + y + z \leq 800 \\ 4x + 2y + 2z \leq 1080 \\ 2x + 3y + z \leq 840 \\ x, y, z \geq 0 \end{cases}$$

(c) Yes, standard max.
Click here for video solution to #9.

(10) The initial and final simplex tableau for the scenario described in problem #9 above are given below. Find the pivot element in the initial tableau and then find and interpret each variable of the final tableau in the context of the problem.

initial tableau

x	y	z	s_1	s_2	s_3	P	const.
3	1	3	1	0	0	0	800
4	2	2	0	1	0	0	1080
2	3	1	0	0	1	0	840
−16	−20	−15	0	0	0	1	0

final tableau

x	y	z	s_1	s_2	s_3	P	const.
7/8	0	1	3/8	0	−1/8	0	195
3/2	0	0	−1/2	1	−1/2	0	260
3/8	1	0	−1/8	0	3/8	0	215
37/8	0	0	25/8	0	45/8	1	7225

Answer:

(a) In the initial tableau, the pivot is in Row 3 column 2

(b) Values of each variable: $x = 0, y = 215, z = 195, s_1 = 0, s_2 = 260, s_3 = 0, P = 7225$

(c) The company will obtain a max value of $7,225 when they manufacture 0 Product A, 215 of Product B, and 195 of Product C. The company will have 260 labor hours in Dept II leftover, but will have used all labor hours in Dept I and Dept III.

(11) Write the initial simplex tableau, if possible, for the linear programming problem. If it is not possible to write the initial simplex tableau, explain why not.

Objective: Maximize $p = \frac{1}{2}x + 3y + z + 4w$ subject to:

$$\begin{cases} x + y + z + w \leq 40 \\ 2x + y - 10 \leq z + w \\ y \geq 5w \\ w, x, y, z \geq 0 \end{cases}$$

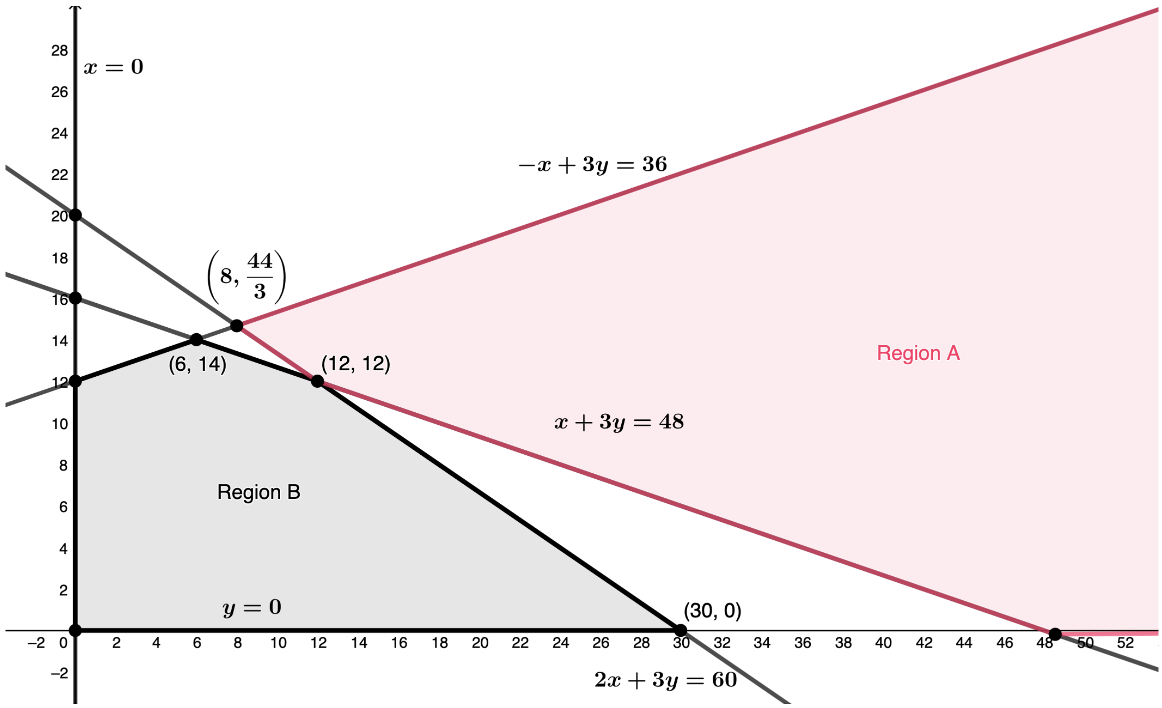
Answer: Yes, standard max.
Objective function: $\frac{-x}{2} - 3y - z - 4w + p = 0$

$$\begin{cases} x + y + z + S_1 = 40 \\ 2x + y - z - w + s_2 = 10 \\ 5w - y + s_3 = 0 \end{cases}$$

initial tableau

x	y	z	w	s_1	s_2	s_3	P	const.
1	1	1	1	1	0	0	0	40
2	1	−1	−1	0	1	0	0	10
0	−1	0	5	0	0	1	0	0
−1/2	−3	−1	−4	0	0	0	1	0

(12) The lines $x = 0, y = 0, -x + 3y = 36, x + 3y = 48$, and $2x + 3y = 60$ are graphed below.



(a) State whether Regions A and B are bounded or unbounded.
Answer: Region A, unbounded. Region B, bounded.

(b) Determine the system of linear inequalities that represent Region A and Region B.
Answer:

$$\text{Region A: } \begin{cases} -x + 3y \leq 36 \\ 2x + 3y \geq 60 \\ x + 3y \geq 48 \\ x, y \geq 0 \end{cases}$$

$$\text{Region B: } \begin{cases} -x + 3y \leq 36 \\ 2x + 3y \leq 60 \\ x + 3y \leq 48 \\ x, y \geq 0 \end{cases}$$

(c) Then find the maximum and minimum values for each region given the objective function $M = 8x + 6y$.
Answer: Region A has a minimum value of 152 and no maximum value.
Region B has a maximum value of 240 and a minimum value of 0.

(13) For the tableau given below, find the corner point at which the tableau currently correspond and then determine if it is the optimal corner? If it is not the optimal corner, give the next pivot element.

x	y	s_1	s_2	s_3	P	const.
$-1/3$	1	$1/3$	0	0	0	12
2	0	-1	1	0	0	12
3	0	-1	0	1	0	24
-4	0	2	0	0	1	72

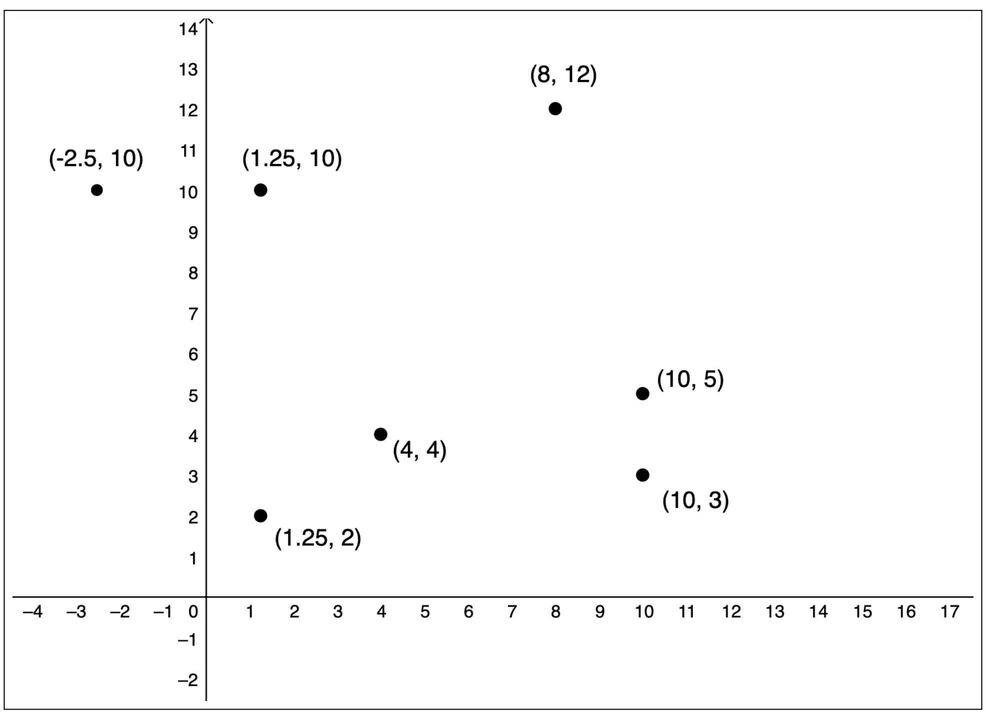
Answer:

(a) Corner point is $(0, 12)$

(b) Not optimal; the next pivot element is 2.

(14) Consider the linear programming problem:

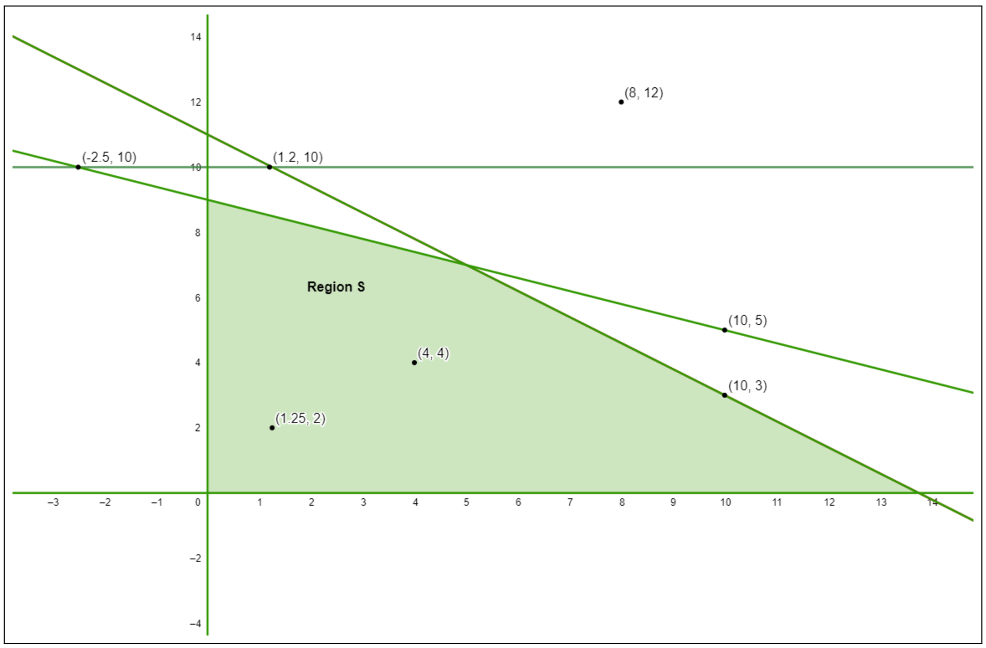
$$\text{Maximize } P = 16x + 20y \text{ subject to } \begin{cases} 4x + 5y \leq 55 \\ 0 \leq y \leq 10 \\ 2x + 5y \geq 45 \\ x \geq 0 \end{cases}$$



(a) Determine if this is a standard maximization problem.
Answer: Not a standard maximization problem because $2x + 5y \geq 45$ cannot be manipulated of the form $a_1x + a_2y \leq V$ where $V \geq 0$.

(b) Use the graph below to graph the constraints for linear programming problem, find the feasible region, and label the feasible region S.

Answer:



(c) Use the method of corners to solve the linear programming problem.

Answer:

(x, y)	$P = 16x + 20y$
$(0, 9)$	180
$(0, 0)$	0
$(13.75, 0)$	220
$(5, 7)$	220

P has a max value of 220 at any point along the line $y = \frac{-4x}{5} + 11$ for $5 \leq x \leq 13.75$.