WIR: Sections 5.3 and 5.4

This document contains the answers to the posed problems. Video solutions will be added as they are produced.

Section 5.3

(1) For the rational functions below state the domain, the $x$-intercept(s), the $y$-intercept, any vertical asymptotes, the coordinates of any holes, and the end behavior.

(a) \( f(x) = \frac{3x^2 - 12x - 15}{x^2 + 2x - 24} \)

\textit{Answer:} Domain: \((-\infty, -6) \cup (-6, 4) \cup (4, \infty)\)

$x$-intercept(s): \((5, 0)\) and \((-1, 0)\)

$y$-intercept: \((0, \frac{5}{8})\)

Vertical Asymptotes: \(x = -6, x = 4\)

Holes: none

End Behavior: As \(x \to \pm \infty, f(x) \to 3\)

(b) \( g(x) = \frac{2x(2x - 1)}{(2x - 1)(x + 3)} \)

\textit{Answer:} Domain: \((-\infty, -3) \cup (-3, \frac{1}{2}) \cup (\frac{1}{2}, \infty)\)

$x$-intercept(s): \((0, 0)\)

$y$-intercept: \((0, 0)\)

Vertical Asymptotes: \(x = -3\)

Holes: \((\frac{1}{2}, \frac{2}{7})\)

End Behavior: As \(x \to \pm \infty, f(x) \to 2\)

(c) \( h(x) = \frac{x^2 + 3x}{(2x - 5)(x + 3)} \)

\textit{Answer:} Domain: \((-\infty, -3) \cup (-3, \frac{5}{2}) \cup (\frac{5}{2}, \infty)\)

$x$-intercept(s): \((0, 0)\)

$y$-intercept: \((0, 0)\)

Vertical Asymptote: \(x = \frac{5}{3}\)

Holes: \((-3, )\) End Behavior: As \(x \to \pm \infty, f(x) \to \frac{1}{2}\)
(d) $p(x) = \frac{3x(x - 1)}{(x - 1)^2(x + 4)}$

Answer: Domain: $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$
x-intercept(s): $(0, 0)$
y-intercept: $(0, 0)$
Vertical Asymptotes: $x = 1, x = -4$
Holes: none
End Behavior: As $x \to \pm\infty$, $f(x) \to 0$

(2) Perform the indicated operation and simplify the result.

(a) $\frac{x - 2}{x + 3} + \frac{x + 1}{x}$

Answer: $\frac{2x^2 + 2x + 3}{x(x + 3)}$

(b) $\frac{2x - 1}{x + 3} - \frac{2x + 5}{2x^2 + 7x + 3}$

Answer: $\frac{2(2x - 3)(x + 2)}{(2x - 3)(x + 3)}$

(c) $\frac{x^2 - 9}{3x^2 - 6x + 3} \cdot \frac{x^2 + 4x - 5}{(x + 3)(x - 3)}$

Answer: $\frac{3(x - 1)(x - 5)}{3(x + 1)(x + 3)}$

(d) $\frac{2x^2 + 16x}{x^3 - 16x} \div \frac{3x^2 + 12x - 96}{x^2 + 6x + 8}$

Answer: $\frac{2(x + 2)}{3(x - 4)^2}$

(e) $\frac{x + 1}{x - 3} - \frac{3}{x + 2} \div \frac{x^2 - 5x + 6}{x^2 - 4}$

Answer: $\frac{x - 2}{x - 3}$

(3) Compute and simplify the difference quotient for

(a) $g(x) = \frac{-2}{x + 3}$

Answer: $\frac{1}{(x + 3)(x + h + 3)}$

(b) $f(x) = 2x^2 - 5x$

Answer: $4x + 2h - 5$

Section 5.4

(4) Simplify the expression $-6x^{-4}y^5(12xy^{-3})^{-2}$. Assume $(x, y > 0)$

Answer: $-y^{11}$

$24x^6$
(5) Rewrite \( g(x) = \frac{4\sqrt[3]{3x^2 + x}}{12} \) is its equivalent exponent form.

\[
\text{Answer: } \frac{1}{3}(3x^2 + x)^{1/5}
\]

(6) Rewrite \( f(x) = 8(4x^2 + 3)^{-2/3} \) equivalent radical form.

\[
\text{Answer: } \frac{8}{\sqrt[3]{(4x^2 + 3)^2}}
\]

(7) State the domain of each function, using interval notation.

(a) \( f(x) = \sqrt[3]{2x + 1} \)

\[
\text{Answer: } (-\infty, \infty)
\]

(b) \( g(x) = \sqrt[3]{2x + 1} \)

\[
\text{Answer: } \left[ -\frac{1}{2}, \infty \right)
\]

(c) \( p(x) = \frac{3}{\sqrt[3]{2x + 1}} \)

\[
\text{Answer: } \left( -\infty, -\frac{1}{2} \right) \cup \left( -\frac{1}{2}, \infty \right)
\]

(d) \( z(x) = \frac{x - 3}{\sqrt[3]{8 - x}} \)

\[
\text{Answer: } (-\infty, 8)
\]

(e) \( n(x) = \frac{x^2 - 3x - 4}{\sqrt[3]{x^2 - 64}} \)

\[
\text{Answer: } (-\infty, -8) \cup (-8, 8) \cup (8, \infty)
\]

(f) \( y(x) = \frac{\sqrt{-2x + 10}}{\sqrt[3]{x^2 - 16}} \)

\[
\text{Answer: } (-\infty, -4) \cup (-4, 4) \cup (4, 5]
\]

(8) Compute and simplify the difference quotient for \( g(x) = -6(x - 4)^{1/2} \).

\[
\text{Answer: } -\frac{6}{\sqrt{x} + h - 4x - 4}
\]