WIR SOLUTIONS: Sections 5.5 and 5.6

This document contains the answers to the posed problems. Video solutions will be added as they are produced.

Section 5.5

(1) Given \( r(x) = \begin{cases} \sqrt{7 - 3x} & x < 0 \\ \frac{x^2 - 2x}{x - 4} & 1 \leq x < 8 \\ (4 - x)^{2/3} & x \geq 8 \end{cases} \), find the following.

(a) the domain of \( r(x) \).
Answer: \((-\infty, 0) \cup (1, 4) \cup (4, \infty)\)

(b) \( r(-1) \)
Answer: \(\sqrt{10} \)

(c) \( r(1) \)
Answer: \(\frac{1}{3} \)

(d) \( r(5) \)
Answer: 15

(e) \( r(2) + r(12) \)
Answer: 4

(2) Find the piecewise-defined function for the graph of \( g(x) \) below.

\[
\begin{cases} 
|x + 4| & x < 3 \\
2 & x \geq 3
\end{cases}
\]
(3) Graph $f(x) = \begin{cases} 
1 - 3x & x < -1 \\
\frac{1}{2}x^2 & -1 < x < 4 \\
3 & x = -1 \\
-2 & x \geq 4 
\end{cases}$

Answer:

(4) Write $h(x) = |3 - 2x|$ as an equivalent piecewise-defined function.

Answer: $h(x) = \begin{cases} 
3 - 2x & x \leq \frac{3}{2} \\
2x - 3 & x > \frac{3}{2} \n\end{cases}$

(5) A local internet provider charges customers a flat rate of $60 per month for the first 500 gigabytes (GB) of usage. If usage exceeds 500 GB per month, the company charges $10 for each additional 50 GB used. Write the function, $B(x)$, which gives the dollar amount of a monthly internet bill when customer uses $x$ GB of internet per month.

Answer: $B(x) = \begin{cases} 
60 & 0 \leq x \leq 500 \\
0.2x - 40 & x > 500 \n\end{cases}$
Section 5.6

(6) For each of the exponential functions below, state (a) whether it is a growth or decay function, (b) the domain, (c) the range, (d) the end behaviors (i.e., behavior of the function values as \(x \to \pm\infty\)), (e) the \(x\)-intercept(s), and (f) the \(y\)-intercept.

(a) \(g(x) = 2\left(\frac{3}{2}\right)^x\)
Answer: (a) \(g(x)\) is an exponential growth function, (b) domain is \((-\infty, \infty)\), (c) range is \((0, \infty)\), (d) As \(x \to \infty, g(x) \to \infty\). As \(x \to -\infty, g(x) \to 0\), (e) no \(x\)-intercept(s), (f) \(y\)-intercept is \((0, 1)\).

(b) \(h(x) = 2\left(\frac{3}{2}\right)^{-x}\)
Answer: (a) \(h(x)\) is an exponential decay function, (b) domain is \((-\infty, \infty)\), (c) range is \((0, \infty)\), (d) As \(x \to \infty, h(x) \to 0\). As \(x \to -\infty, h(x) \to \infty\), (e) no \(x\)-intercept(s), (f) \(y\)-intercept is \((0, 1)\).

(c) \(t(x) = -2e^x\)
Answer: (a) \(t(x)\) is an neither an exponential growth or decay function, (b) domain is \((-\infty, \infty)\), (c) range is \((-\infty, 0)\), (d) As \(x \to \infty, t(x) \to -\infty\). As \(x \to -\infty, t(x) \to 0\), (e) no \(x\)-intercept(s), (f) \(y\)-intercept is \((0, -2)\).

(d) \(z(x) = -2e^{-x}\)
Answer: (a) \(z(x)\) is an neither an exponential growth or decay function, (b) domain is \((-\infty, \infty)\), (c) range is \((-\infty, 0)\), (d) As \(x \to \infty, z(x) \to 0\). As \(x \to -\infty, z(x) \to -\infty\), (e) no \(x\)-intercept(s), (f) \(y\)-intercept is \((0, -2)\).

(7) Find the domain of each of the following functions.

(a) \(f(x) = \frac{e^{x+2}}{\sqrt{x+4}}\)
Answer: \((-\infty, -4) \cup (-4, \infty)\)

(b) \(g(x) = \frac{e^{x+2}}{\sqrt{x+4}}\)
Answer: \((-\infty, -4) \cup (-4, -2) \cup (-2, \infty)\)

(c) \(h(x) = \frac{2\sqrt{5-2x}}{3x-1}\)
Answer: \((-\infty, \frac{5}{2}]\)
(8) Solve each of the following equations for \( x \).

(a) \[ 25^{-2x} \cdot \frac{2}{125^{x+4}} = 250 \]

\[ Answer: \ x = -\frac{15}{7} \]

(b) \[ \frac{49^{3x}}{7^{x^2+1}} = 7^3 \cdot 7^4 \]

\[ Answer: \ x = 2, 4 \]

(9) In 25 years you want to have $150,000 in a bank account. You found a bank that will guarantee 4.8% interest, compounded continuously as long as there are no withdraws after you open the account. How much should you invest now (to the nearest cent) to have the $150,000 in 25 years?

\[ Answer: \ $45,179.13 \]