Problem 1. A sports store in town found that when the price of a tennis racquet is $81, the number of racquets bought per month is 4900 while the number of racquets supplied by the manufacturer is 5250. When the price of racquets drops by $9, the demand increases by 900 racquets and the manufacturer will only supply 5100 racquets at this price.

1. Find the linear demand equation.
2. At what price will no consumers be willing to buy racquets?
3. According to the above model, how many racquets would consumers buy if they were available at a price of $0?
4. Find the linear supply equation
5. At what price will suppliers be unwilling to supply racquets?

(6) Find equilibrium price & quantity

\[ y = mx + b \]
\[ m, b \]
\[ \text{new price} = 81 - 9 = 72 \]

\[ (4900, 81) \]
\[ (5800, 72) \]
\[ (5250, 81) \]
\[ (5100, 72) \]

\[ m = \frac{81 - 72}{4900 - 5800} = \frac{9}{-900} = -\frac{1}{100} \]

\[ y - 81 = \left(-\frac{1}{100}\right)(x - 4900) \]

\[ y = p(x) = \frac{-x}{100} + 49 + 81 \]

\[ p(x) = \frac{-x}{100} + 130 \]

\[ 900 + 4900 \]

\[ y = \left(\frac{3}{50}\right)(x - 5250) \]

\[ y = p(x) = \frac{3x}{50} - 234 \]

\[ p(x) = \frac{3x}{50} - 234 \]

5. What price?

\[ y \text{ intercept } (x=0) \]

\[ \$ - 234 \]
2. $y$ intercept: $(x=0)$

$p(0) = 0 + 130$

$\text{Ans: } \$130$

3. $x$-intercept $(y=0)$

$p(x) = 0 = \frac{-x}{100} + 130$

$\frac{x}{100} = 130$

$\text{Ans. } X = 13000$

\[ S(x) = -234\]

6. $D(x) = S(x)$

Solve for $x$ or equilibrium quantity.

Plug in $x$ into $D(x)$ or into $S(x)$ to find price.
Problem 2. A construction company purchased a crane in 1996 at a cost of $55,000 and has a scrap value of $0 after 25 years. If the machine depreciates linearly over time, find

1. the rate of depreciation
2. the linear equation that shows the book value of the crane after $t$ years
3. the value of the crane in the year 2000.

1996 $\rightarrow$ 550000 $\Rightarrow$ (0, 550000)

Set this to time $t=0$

\[ m = \frac{55000 - 0}{0 - 25} = -\frac{55000}{25} = -2200 \]

2. \[ v(t) = mt + b \]

\[ v(0) = -2200(0) + 55000 = 55000 \]

(2) \[ v(t) = -2200t + 55000 \]

1. The rate of depreciation is $2200 per year.

3. In the year 2000

\[ t=0 \quad (1996) \]

\[ t=4 \quad (2000) \]

\[ v(4) = (-2200)(4) + 55000 = 46,200 \]
Problem 3. The fixed cost for a company that makes waffles is $12,000 and it costs $50 to make each unit. The waffle company must sell a minimum of 240 waffle machines to no longer be losing money on any given day. Find

1. the linear cost function.
2. the price per item that each waffle machine is sold at.
3. the revenue function.
4. the total revenue when 500 machines are sold.
5. the profit function.
6. the total profit when 500 machines are sold.

\[ y = mx + b \]  
Fixed costs.

\[ 240 = x \]

\[ 12000 = b \]

\[ 50 = m \]

\[ CC(x) = 50x + 12000 \]

\[ \begin{align*}
    & x = 240 \\
    & C(240) = R(240) \\
    & \text{At break-even point:} \quad P(x) = 0 \\
    & \Rightarrow R(x) = C(x) \\
    & 50(240) + 12000 = P(240) \\
    & \text{Solve for } p \\
    & 24000 = (240)p \\
    & \frac{100}{240} = p \\
    & \Rightarrow p = \text{selling price/item} = $100
\end{align*} \]

\[ R(x) = px = 100x \]
(4) \( R(500) = 100 \times 500 = \$50000 \)

(5) \[ P(x) = R(x) - C(x) \]
\[ = 100x - (50x + 12000) \]
\[ P(x) = 50x - 12000 \]

(6) \[ P(500) = 50 \times 500 - 12000 = \$13000 \]
Problem 4. An investment club has $500,000 earmarked for investment in stocks. To arrive at an acceptable overall level of risk, the stocks have been classified into three categories: high-risk, medium-risk, and low-risk. Management estimates that high-risk stocks will have a rate of return of 16% per year, medium-risk stocks will have a rate of return of 10% per year and low-risk stocks, 4% per year. The members have decided that the investment in medium-risk stocks should be equal to the sum of the investment in the other two categories. How much should the club invest in each type of stock if the investment goal is to have a return of 9% (= $45,000) per year on the total investment?

Define your variables

\[ 9 \times \frac{500,000}{100} = 45,000 \]

Target return

Define your variables

\[ x = \text{money invested in high-risk stocks} \]
\[ y = \text{money invested in medium-risk stocks} \]
\[ z = \text{money invested in low-risk stocks} \]

Return investment ratio

\[ 0.16x + 0.10y + 0.04z = 45,000 \]

Investment ratio

\[ x + y + z = 500,000 \]

\[ y = x + z \]

\[ x - y + z = 0 \]

\[ \begin{bmatrix} 0.16 & 0.1 & 0.04 & 45000 \\ 1 & 1 & 1 & 500000 \\ 1 & -1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} \]

Investment in high-risk stock

\[ x = 83,333.33 \]

Investment in medium-risk stock

\[ y = 250,000 \]
\[
\begin{align*}
\text{Given: } & & \\
& & \\
\text{ } & & v \quad \\
\left\{ & & \\
& & \text{medium} \\
& & \right. \\
\text{ } & & \\
& & \\
& & \\
\frac{y}{z} = \$250,000 \\
& & \\
& & (\text{low}) \\
\frac{y}{z} = \$166,666.67 \\
& & \\
& & (\text{low}) \\
\end{align*}
\]
Problem 5. Given the following matrices,

\[
A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & a \\ 1 & 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & 1 \\ a & 6 & b \\ 2 & b & 5 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & -10 & 5 \\ 11 & 24 & 11 \\ 0 & -19 & 14 \end{bmatrix}.
\]

If \( AB = C^T \), find the values of \( a \) and \( b \).

\[
\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & a \\ 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ a & 6 & b \\ 2 & b & 5 \end{bmatrix}
\]

\[
= \begin{bmatrix} 3 + 2a + 2 & 2 + 12 + b & 1 + 2b + 5 \\ 3a + 2a & 18 + ab & 3b + 5a \\ 3 + 4a + 10 & 2 + 24 + 5b & 1 + 4b + 25 \end{bmatrix}
\]

\[
= \begin{bmatrix} 5 + 2a & 14 + b & 6 + 2b \\ 5a & 18 + ab & 3a + 3b \\ 13 + 4a & 26 + 5b & 26 + 4b \end{bmatrix} = \begin{bmatrix} 1 & 11 & 0 \\ -10 & 24 & -19 \\ 5 & 11 & 14 \end{bmatrix}
\]

\[
5 + 2a = 1 \\
2a = 1 - 5 = -4 \\
\text{Ans: } a = -2
\]

\[
6 + 2b = 0 \\
2b = -6 \\
\text{Ans: } b = -3
\]
Problem 6. Which of the following augmented matrices is in row reduced form? If the augmented matrix is not in a row reduced form, how will you convert it to a row reduced form? Does the row reduced form correspond to a unique solution, no solution or infinite solutions? Rewrite the augmented matrices in equation form and give the solution.

\[
\begin{bmatrix}
1 & 0 & 0 & 5 \\
0 & 1 & 0 & 3 \\
0 & 0 & 0 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 2
\end{bmatrix}
\Rightarrow \text{Independent System.}
\]

\(2) \quad \text{No!}

\[
\frac{1}{2} R_3 \rightarrow R_3
\]

\[
R_1 \rightarrow R_1
\]

Solution:
\(x = 3, \quad y = 3, \quad z = 2.\)

\((x, y, z) = (3, 3, 2) \rightarrow \text{One solution}\)

\[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & 3 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & \frac{1}{2} \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & \frac{3}{2}
\end{bmatrix}
\]

\(\text{No!}\)

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 3
\end{bmatrix}
\]

\(\text{Yes}\)
\begin{align*}
&\text{(3)} \quad \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \\
&\text{Independent system.}
\\
&\text{Ans: } (x, y, z) = (-1, 2, 3)
\\
&\text{(4)} \quad \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_3} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}
\\&\text{NO}
\\&\text{Yes!}
\\&x \neq y
\\&R_1: \quad x + 2z = 0 \quad ? \quad \text{Dependent system}
\\&R_2: \quad y + 4z = 1 \quad \text{Let } w = t
\\&R_3: \quad z + 3w = 1 \quad \text{then } x = -2w = -2t
\\&\text{Parametric sol: } (x, y, z, w) = (-2t, 1-4t, 1-3t, t)
Parametric soln: \((x, y, z, w) = (-2t, 1-4t, 1-2t, t)\)

\[
\begin{bmatrix}
1 & 0 & 0 & 7 \\
0 & 0 & 1 & 3 \\
0 & 1 & 0 & 3
\end{bmatrix}
\]

\(R_2 \leftrightarrow R_3\)

Eqns:
- \(R_1: x = 7\)
- \(R_2: y = 3\)
- \(R_3: 0 = 1 \leftarrow \text{not true!}\)

And: system is inconsistent: \(\text{No soln}\)

Recap:
1. Independent system: unique value for each variable.
2. Inconsistent system: \(0 = 1\) or \(5 = 3\) etc.
3. Dependent system: 2 variables only show up together, usually happens when \(0 = 0\)!
Problem 7. Solve the following system of linear equations and classify it. In case of a dependent system, present your answer in parametric form.

(1) \[ \begin{align*}
20x - 16y &= 72 \\
5x - 4y &= 18 \\
8y - 10x + 36 &= 0
\end{align*} \]

\[ \Rightarrow \begin{align*}
20x - 16y &= 72 \\
5x - 4y &= 18 \\
-10x + 8y &= -36
\end{align*} \]

\[ \begin{bmatrix}
20 & -16 & 72 \\
5 & -4 & 18 \\
-10 & 8 & -36
\end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix}
1 & -4/5 & 18/5 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \Rightarrow \text{Dependent system}
\]

Let \( y = t \)

then \( \frac{-4}{5}y = \frac{18}{5} \)

\[ x = \frac{4}{5}t + \frac{18}{5} \]

And \( (x, y) = \left( \frac{4}{5}t + \frac{18}{5}, t \right) \)

(2) \[ \begin{align*}
2x + 2y + z &= 7 \\
2x &= 3 + y + 3z \\
3y + 2z &= 4
\end{align*} \]

\[ \begin{align*}
2x + 2y - 2z &= 7 \\
2x - y - 3z &= 3 \\
0x + 3y + 2z &= 4
\end{align*} \]

\[ \begin{bmatrix}
2 & 2 & -1 & 7 \\
2 & -1 & -3 & 3 \\
0 & 3 & 2 & 4
\end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix}
1 & 0 & -7/6 & 13/6 \\
0 & 1 & -7/3 & 4/3 \\
0 & 0 & 0 & 0
\end{bmatrix} \]

\[ x - \frac{7}{6}z = \frac{13}{6} \quad \Rightarrow \quad \text{Dependent} \]
\[ \begin{align*}
&x - \frac{7}{6}z = \frac{13}{6} \\
&y + \frac{2}{3}z = 4/3 \\
&0 = 0
\end{align*} \]

Dependent System

Let \( z = t \)

\[ \text{Soln:} (x, y, z) = \left( \frac{7}{6}t + \frac{13}{6}, \quad -\frac{2}{3}t + \frac{4}{3}, \quad t \right) \]

Parameteric Soln
Problem 8. Pivot the augmented matrix about row 2, column 2. Do not completely reduce the matrix to reduced row echelon form. Specify what row operation is being performed in each step, using the correct notation.

\[
\begin{bmatrix}
1 & 4 & -2 & 1 \\
0 & 3 & -9 & 12 \\
0 & -2 & 3 & -7 \\
\end{bmatrix}
\]

\[
\begin{array}{c}
\frac{1}{3} R_2 \rightarrow R_2 \\
\end{array}
\]

\[
\begin{bmatrix}
1 & 4 & -2 & 1 \\
0 & 1 & -3 & 4 \\
0 & -2 & 3 & -7 \\
\end{bmatrix}
\]

\[
\begin{array}{c}
R_1 + 2R_3 \rightarrow R_1 \\
\end{array}
\]

\[
\begin{bmatrix}
1 & 0 & 4 & -13 \\
0 & 1 & -3 & 4 \\
0 & -2 & 3 & -7 \\
\end{bmatrix}
\]

\[
\begin{array}{c}
R_3 + 2R_2 \rightarrow R_3 \\
\end{array}
\]

\[
\begin{bmatrix}
1 & 0 & 4 & -13 \\
0 & 1 & -3 & 4 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

augmented matrix is pivoted around row 2, column 2.
Problem 9.
\[
\begin{align*}
4x - 2y &= 5 \\
9x + ky &= c
\end{align*}
\]

\[\Rightarrow \quad \begin{align*}
y &= 2x - \frac{5}{2} \\
k\y &= -9x + c \\[y &= \frac{-9}{k}x + \frac{c}{k}
\end{align*}\]

1. Find the value of \( k \) for which the given system of equations has no solution.
2. What would be the values of \( k \) and \( c \) for the given system to have infinite solutions?

\( \frac{1}{2} \) No solution when slopes are same value

\[
m_1 = m_2
\]

\[
2 = \frac{-9}{k} \quad \text{solve for } k.
\]

\[
k = -\frac{9}{2}
\]

System has no solutions when \( k = -\frac{9}{2} \)

\( \frac{2}{2} \) Infinite solutions \( \Rightarrow \) overlapping lines

slopes are same \( (k = -\frac{9}{2}) \)

\[
y \text{ intercepts are same}
\]

\[
b_1 = b_2
\]

\[
-\frac{9}{2} = \frac{c}{k}
\]

\[
-\frac{5}{2} = \frac{c}{(-\frac{9}{2})} = -\frac{2c}{9} \quad \text{solve for } c
\]

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\[ c = \frac{(-5)(1)}{(2)(-2)} = \frac{45}{4} \]
Problem 10. Given the matrices $A, B, C, D$ below, evaluate the following:

$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 1 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 3 & 1 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 6 & 1 \\ 2 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 8 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

(1) $A - 5B$
(2) $B + D$
(3) $D + C^T$

\[ \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 1 & 0 & 5 \end{bmatrix} - \begin{bmatrix} -10 & 15 & 5 \\ 0 & 5 & 0 \\ 0 & -15 & 5 \end{bmatrix} = \begin{bmatrix} 11 & -16 & -5 \\ 0 & -2 & 2 \\ 1 & 15 & 0 \end{bmatrix} \]

(2) $B + D$ is not possible because the matrices do not have the same size

\[ \begin{bmatrix} 2 & 8 & 0 \\ 0 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 14 & 2 \\ 0 & 6 & 4 \end{bmatrix} \]