Week-in-Review 6, (Chapter 3.4, 4.1, 4.2)

Problem 1. Is the following linear programming problem a standard maximization problem? If not, state why not. If yes, set up the initial simplex tableau corresponding to the given linear program.

(1) Objective: Maximize $P = 20x + 45y$
Subject to:
$y \geq x - 8$
$6y \geq 4x$
$x \geq 0, \ y \geq 0$
(2) Objective: Maximize $P = 4x + 7y$

Subject to: $x + 3y \leq 0$
$6x + 2y \leq 12$
$x \leq 0, \ y \leq 0$
(3) Objective: Maximize $P = 17x + 15y$

Subject to:
\[ \frac{1}{7}x + \frac{3}{8}y \leq 42 \]
\[ 9 \geq 3x + 6y \]
\[ x - y \leq 283 \]
\[ x \geq 0, \ y \geq 0 \]
Problem 2. For the given simplex tableau, if the simplex tableau is the final tableau, state the optimal solution. If the tableau is not the final tableau, state the value of each variable and whether the variable is a basic or non-basic variable, the pivot column, the pivot row and the pivot element. If the Method of corners had been used, state the corner point corresponding to the pivot element for that simplex tableau.

\[
\begin{bmatrix}
  x & y & z & s_1 & s_2 & s_3 & P \\
  6 & 0 & 2 & 1 & 0 & 0 & 0 & 120 \\
  0 & 1 & 4 & 0 & 1 & 0 & 0 & 84 \\
  7 & 8 & 0 & 0 & 0 & 1 & 0 & 100 \\
  -6 & -10 & -13 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]
\[
\begin{bmatrix}
    x & y & z & s_1 & s_2 & s_3 & P \\
    0 & 0 & -\frac{1}{20} & 1 & -\frac{8}{5} & -\frac{4}{35} & 0 & 230 \\
    0 & 1 & 4 & 0 & 8 & 2 & 0 & 2000 \\
    1 & 0 & 6 & 0 & 8 & 3 & 0 & 2480 \\
    0 & 0 & 22 & 0 & 72 & 17 & 1 & 0
\end{bmatrix}
\]
Problem 3. A linear programming problem was solved using the Simplex Method. All tableaus are given, in order, below. If the Method of Corners had been used, state the corner point corresponding to each tableau.

\[
\begin{bmatrix}
   x & y & s_1 & s_2 & s_3 & P \\
   2 & 6 & 1 & 0 & 0 & 0 & 12 \\
   1 & 0.25 & 0 & 1 & 0 & 0 & 25 \\
   3 & 0.5 & 0 & 0 & 1 & 0 & 60 \\
   -2 & -3 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

Write the linear programming problem corresponding to this simplex tableau.

\[
\begin{bmatrix}
   x & y & s_1 & s_2 & s_3 & P \\
   \frac{1}{3} & 1 & \frac{1}{6} & 0 & 0 & 0 & 2 \\
   \frac{11}{12} & 0 & -\frac{1}{24} & 1 & 0 & 0 & \frac{49}{2} \\
   \frac{17}{6} & 0 & -\frac{1}{12} & 0 & 1 & 0 & 59 \\
   -1 & 0 & \frac{1}{2} & 0 & 0 & 1 & 6
\end{bmatrix}
\]

\[
\begin{bmatrix}
   x & y & s_1 & s_2 & s_3 & P \\
   1 & 3 & \frac{1}{2} & 0 & 0 & 0 & 6 \\
   0 & -\frac{11}{4} & -\frac{1}{2} & 1 & 0 & 0 & 19 \\
   0 & -\frac{17}{2} & -\frac{3}{2} & 0 & 1 & 0 & 42 \\
   0 & 3 & 1 & 0 & 0 & 1 & 12
\end{bmatrix}
\]
Problem 4. A furniture manufacturer produces chairs, sofas, and ottomans. The chairs require 5 square feet of wood, 1 pound of foam rubber, and 10 square yards of fabric. The sofas require 35 square feet of wood, 2 pounds of foam rubber, and 20 square yards of fabric. The ottomans require 9 square feet of wood, 0.2 pounds of foam rubber, and 10 square yards of fabric. The manufacturer has 405 square feet of wood, 25 pounds of foam rubber, and 410 square yards of fabric in stock. If the chairs yield a profit of $300, the sofas $200, and the ottomans $220 each, how many of each should be produced to maximize the profit? Are any materials leftover in this case?
Problem 5. Consider the following experiment: First, a card is drawn from a standard 52-card deck and the suit is recorded. Next, a fair 3-sided die is rolled and the number rolled is recorded.

(1) Write the sample space for this experiment.

(2) State the total number of possible events of the sample space.

(3) Write the outcomes in the event B, a number greater than 2 is rolled.

(4) Write the outcomes in the event C, a number greater than 3 is rolled.

(5) Write the outcomes in the event D, a red card is drawn.

(6) Write the outcomes in the event E, a 2 is rolled and a clubs is drawn.

(7) Write the outcomes in the event F, a 2 is rolled or a clubs card is drawn.

(8) Which of the events from parts (c)-(f) are simple events, if any?
Problem 6. Shade the following regions in the given Venn Diagram

(1) $A \cup B^C$

(2) $(A \cup B)^C$

(3) $A \cap B^C$
(4) $A \cap B^C$

(5) $(A \cap B)^C$

(6) $A^C \cap B^C$
Problem 7. A fair coin is tossed four times and the side landing up on each toss is noted.

(1) Draw a tree diagram for this experiment.

(2) Write down the sample space for this experiment.

(3) Construct a probability distribution for this experiment.

(4) What is the probability that the first toss shows "tails"?
(5) What is the probability that the last toss shows tails?

(6) What is the probability that the coin shows no tails?

(7) What is the probability that the coin shows tails exactly once?

(8) What is the probability that the coin shows tails at least once?

(9) What is the probability that the coin shows tails no more than 2 times?