Problem 1. If $F$ and $G$ are two events in an experiment with $P(F) = 0.44$, $P(G) = 0.64$ and $P(F \cap G) = 0.38$, calculate the following probabilities.

a. $P(F^c) = 1 - P(F) = 1 - 0.44 = 0.56$

b. $P(G^c) = 1 - P(G) = 1 - 0.64 = 0.36$

c. $P(F \cup G) = 0.06 + 0.38 + 0.26 = 0.7$

d. $P(F \cap G^c) = 0.06$

e. $P(F^c \cap G^c) = 0.3$

$P(F \cap G^c)$

- A belongs to $F$ but not $G$. $P(F \cap G^c)$
\textbf{AND} : \cap

\textbf{OR} : \cup

\textbf{NOT} : \bar{\lambda}c
Problem 2. In a class of 40 students, 17 are taking math, 31 are taking English and 15 are taking both math and English. What is the probability that a student chosen at random is taking

a. math or English

\[
P(\text{MUE}) = \frac{2+15+16}{40} = \frac{33}{40}
\]

OR: \[P(\text{M}) + P(\text{E}) - P(\text{M\&E}) = \frac{17+31-15}{40} = \frac{33}{40}\]

b. math and English

\[
P(\text{M\&E}) = \frac{15}{40}
\]

c. math but not English \[\Rightarrow \text{X region}\]

\[
P(\text{M\&E}^c) = \frac{2}{40}
\]

d. English but not math \[\Rightarrow \text{Z region}\]

\[
P(\text{M}^c \& E) = \frac{16}{40}
\]

e. neither math nor English \[\Rightarrow \omega \text{ region}\]

\[
P(\text{M}^c \& E^c) = \frac{7}{40}
\]

OR \[P(\text{MUE})^c\]

\[\Rightarrow A^c \cap B^c = (A \cup B)^c\]
Problem 3. An experiment consists of randomly selecting a card from a standard, well-shuffled deck of 52 cards. What is the probability that a card chosen at random is

a. a black card

\[ P(\text{Black}) = \frac{26}{52} \]

b. a red face card

\[ P(\text{Red}) = \frac{2 \times 3}{52} = \frac{6}{52} \]

\[ P(\text{Clubs or an Ace}) = \frac{13}{52} \]

\[ P(\text{A}) = \frac{4}{52} \]

\[ P(\text{Clubs or an Ace}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} \]

\[ P(\text{Black 10}) = \frac{2}{52} \]

\[ P(\text{Diamonds and a Face Card}) = \frac{3}{52} \]
Problem 4. Compute the following probabilities based on the probability distribution table given below.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X)$</td>
<td>$\frac{1}{20}$</td>
<td>$\frac{3}{20}$</td>
<td>$\frac{5}{20}$</td>
<td>$\frac{4}{20}$</td>
<td>$\frac{7}{20}$</td>
</tr>
</tbody>
</table>

Is this PD valid?

$$\frac{1}{20} + \frac{3}{20} + \frac{5}{20} + \frac{4}{20} + \frac{7}{20} = 1$$

\[ \therefore \text{Yes.} \]

a. $P(5) = \frac{7}{20}$

b. $P(X > 3)$ (does not include 3)

$$= P(4) + P(5) = \frac{4}{20} + \frac{7}{20} = \frac{11}{20}$$

c. $P(X \leq 4)$

$$= P(1) + P(2) + P(3) + P(4) = 1 - P(5) = \frac{13}{20}$$

d. $P(1 \leq X \leq 5)$

$$= P(2) + P(3) + P(4) + P(5)$$

$$= 1 - P(1) = \frac{17}{20}$$

e. The expected value $E(X)$.

$$E(X) = (1)\left(\frac{1}{20}\right) + (2)\left(\frac{3}{20}\right) + (3)\left(\frac{5}{20}\right) + (4)\left(\frac{4}{20}\right) + (5)\left(\frac{7}{20}\right)$$

$$= \frac{1}{20} + \frac{6}{20} + \frac{15}{20} + \frac{16}{20} + \frac{35}{20}$$

$$= \frac{73}{20}$$

$$= 3.65$$
Problem 5. You pay $8 to play a game where you draw a numbered ball from a bucket containing twelve identical balls numbered 1 – 12. If you draw a ball numbered less than 3, you win a dollar amount equal to the number on the ball. If you draw a ball numbered with a multiple of 3, you win $8. If you draw a ball with any other number, you win nothing. Let X be your winnings and Y be your net winnings.

a. Create the probability distribution for X.

<table>
<thead>
<tr>
<th>Draw</th>
<th>X</th>
<th>Draw</th>
<th>Y</th>
<th>Draw</th>
<th>Rest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-3</td>
<td>2</td>
<td>2-3</td>
<td>3, 4, 9, 12</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-2</td>
<td></td>
<td>-1</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Prob. Dist. (P(X))

\[
\begin{array}{c|c|c|c|c}
\text{Draw} & \text{X} & \text{Y} & \text{Prob.} \\
\hline
1 & 1-3 & 2 & \frac{1}{12} & \frac{1}{12} \\
2 & 2-3 & \frac{3}{12} & \frac{3}{12} \\
3, 4, 9, 12 & 3 & \frac{4}{12} & \frac{4}{12} \\
0 & 0 & \frac{6}{12} & \frac{6}{12} \\
\end{array}
\]

b. What are your expected winnings?

\[
E(X) = \left(1\right)\left(\frac{1}{12}\right) + \left(2\right)\left(\frac{1}{12}\right) + \left(3\right)\left(\frac{4}{12}\right) + \left(0\right)\left(\frac{6}{12}\right) \\
E(X) = \frac{1}{12} - \frac{2}{12} + \frac{3}{12} = \frac{3}{12} = 2.5 \\
\text{net winnings} = \text{earn} - \text{pay} \\
\]

\[
E(X) = 2.5 \\
E(Y) = 2.5 - 8 = -5.5 \\
\]

c. What are your expected net winning?

\[
E(Y) = \left(-2\right)\left(\frac{1}{12}\right) + \left(-1\right)\left(\frac{1}{12}\right) + \left(5\right)\left(\frac{4}{12}\right) + \left(-3\right)\left(\frac{6}{12}\right) \\
E(Y) = -\frac{2}{12} - \frac{1}{12} + \frac{20}{12} - \frac{18}{12} = -\frac{1}{12} = -0.08 \\
\]

d. Is this a fair game?

Ans: No!

For a fair game, \[E(X) = \frac{3}{8}\] or \[E(Y) = 0\]

\[
E(Y) = E(X) - \text{pay} \\
E(Y) = 2.5 - 8 = -5.5 \\
\]
\[ = 2.92 - 3 \]
\[ = -0.08 \]
Problem 6. A bowl consists of one red marble, one green marble, and one yellow marble. John randomly selects a sample of two marbles from the bowl. If there are no green marbles in his sample, he pays Craig $3. Otherwise, Craig pays John $A$. What value of $A$ makes this game a fair one?

Let, $X =$ what Craig earns

<table>
<thead>
<tr>
<th>$X$</th>
<th>$\text{no green}$</th>
<th>$\text{one green}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>3</td>
<td>$-A$</td>
</tr>
<tr>
<td>$p(x)$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
</tbody>
</table>

For a fair game, $E(X) = 0$.

$$E(X) = 3 \left( \frac{1}{3} \right) + (-A) \left( \frac{2}{3} \right) = 0$$

$$\frac{3}{3} - \frac{2A}{3} = 0 \implies 1 = \frac{2A}{3} \implies A = \frac{3}{2}$$

$\therefore A = \frac{3}{2} = \$1.50$
**Problem 7.** You purchase a $250,000 long-term disability policy. The insurance premium for this 1-year policy is $4500. Based on your work environment, the probability of a long-term disability is 0.7%. Let $X$ be the insurance company’s net gain or loss on the policy described. Compute the company’s expected net gain on this policy.

\[
\begin{array}{c|c|c}
\text{disability} & & \text{healthy} \\
\hline
X & 4500 - 250000 \quad = -246,500 & 4500 \\
\hline
p(X) & 0.007 & 1 - 0.007 \quad = 0.993 \\
\end{array}
\]

\[
E(X) = (-246500)(0.007) + (4500)(0.993) = \$2750
\]
Problem 8. A company manufactures small and large picture frames. A small picture frame sells for $2.00 and requires 1 unit of glass and 3 units of metal to make. A large picture frame sells for $3.50 and requires 2 units of glass and 4 units of metal to make. If the company has 100 units of glass and 400 units of metal available, and they want to make no more than four times as many large picture frames as small picture frames, how many picture frames of each size should the company manufacture if they want to maximize their revenue? Set up but do not solve.

**Variables**

\[ x = \text{number of small frames} \]
\[ y = \text{number of large frames} \]

**Objective**

Maximize: \[ R = 2x + 3.5y \]

**Constraints**

\[ x + 2y \leq 100 \quad \rightarrow \text{glass} \]
\[ 3x + 4y \leq 400 \quad \rightarrow \text{metal} \]
\[ y \leq 4x \quad \rightarrow \text{ratio} \]
\[ x \geq 0, \; y \geq 0 \quad \rightarrow \text{non negativity} \]
Problem 9. Given the constraints:

\[
\begin{align*}
8x - 5y &\leq 40 \\
3x + 2y &\geq 12 \\
x &\geq 0 \\
0 &\leq y \leq 8
\end{align*}
\]

\[\rightarrow 8x - 5y = 40 \quad \& \quad (0, 8) \& \quad (5, 0)\]
\[\rightarrow 3x + 2y = 12 \quad \& \quad (0, 6) \& \quad (4, 0)\]
\[\rightarrow y = 0 \quad (dashed) \quad \& \quad y = 8 \quad (solid)\]

a. Graph the system of linear inequalities. Identify the solution set with an \( S \).
b. Determine if the solution set is bounded or unbounded.
c. State the exact corner points of the solution set.
d. Objective: Maximize \( P = 4x + 7y \)

Test point \( (0, 0) \):
\[8x - 5y \leq 40 \quad 0 \leq 40 \rightarrow \text{true}\]
\[3x + 2y \geq 12 \quad 0 \geq 12 \rightarrow \text{false}\]

Corner points:
A: \((0, 6)\)
B: \((0, 8)\)
C: \((10, 8)\)
D: \((5, 0)\)
E: \((4, 0)\)

d) Corner point | \( P = 4x + 7y \)  
---|---
A: \((0, 6)\) | \( P(A) = 42 \)
B: \((0, 8)\) | \( P(B) = 56 \)
C: \((5, 0)\) | \( P(C) = 10 + 2 \cdot 5 = 30 \)
\( (0, 5) \quad \therefore \quad y = 5 \)

\[ c \, (10, 8) \quad P(c) = 40 + 56 = 96 \]

\[ d \, (6, 0) \quad P(d) = 20 \]

\[ e \, (4, 0) \quad P(e) = 16 \]

Maximum value of \( P \) is 96 at \( (x, y) = (10, 8) \)

\[
\begin{align*}
8x - 5y &= 40 \\
3x + 2y &= 12
\end{align*}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
\begin{bmatrix}
8 & -5 \\
3 & 2
\end{bmatrix}
\begin{bmatrix}
c
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\frac{140}{31} \\
\frac{-24}{31}
\end{bmatrix}
\]
**Problem 10.** The following is a simplex tableau for a linear programming problem.

Is this a final solution? What are the basic and non basic variables and their values? What is the maximum value of P and what is the corner point that it corresponds to?

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>s₁</th>
<th>s₂</th>
<th>s₃</th>
<th>P</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>20</td>
<td>0</td>
<td>210</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-10</td>
<td>0</td>
<td>102</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-60</td>
<td>0</td>
<td>40</td>
<td>-40</td>
<td>1</td>
<td>380</td>
<td></td>
</tr>
</tbody>
</table>

a) final solve? No → negs in bottom row.

b) **Basic vars:**

\[
\begin{align*}
x &= 210 \\
y &= 102 \\
s₁ &= 120 \\
P &= 380
\end{align*}
\]

c) Maximum P is 380 for corner point \((x, y, z) = (210, 102, 0)\).

d) What is the pivot element?

3 in \(R₃C₃\)