# Problem Statements

## Fundamental Trigonometric Identities

**Reciprocal Identities**

\[
csc(\theta) = \frac{1}{\sin(\theta)} \quad \sec(\theta) = \frac{1}{\cos(\theta)} \quad \cot(\theta) = \frac{1}{\tan(\theta)}
\]

**Quotient Identities**

\[
\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}
\]

**Pythagorean Identities**

\[
\sin^2(\theta) + \cos^2(\theta) = 1 \quad 1 + \cot^2(\theta) = \csc^2(\theta) \\
\tan^2(\theta) + 1 = \sec^2(\theta)
\]

**Cofunctions**

\[
\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta) \\
\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta) \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta) \\
\sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta) \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta)
\]

**Even/Odd Identities**

\[
\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta) \\
\csc(-\theta) = -\csc(\theta) \quad \sec(-\theta) = \sec(\theta) \quad \cot(-\theta) = -\cot(\theta)
\]

**Sum and Difference Identities**

\[
\sin(u + v) = \sin(u) \cos(v) + \cos(u) \sin(v) \quad \sin(u - v) = \sin(u) \cos(v) - \cos(u) \sin(v) \\
\cos(u + v) = \cos(u) \cos(v) - \sin(u) \sin(v) \quad \cos(u - v) = \cos(u) \cos(v) + \sin(u) \sin(v) \\
\tan(u + v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u) \tan(v)} \quad \tan(u - v) = \frac{\tan(u) - \tan(v)}{1 + \tan(u) \tan(v)}
\]

**Double Angle Identities**

\[
\sin(2u) = 2\sin(u) \cos(u) \quad \cos(2u) = \cos^2(u) - \sin^2(u) = 2\cos^2(u) - 1 = 1 - 2\sin^2(u)
\]
1. Verify the following identities
   a. \( \frac{3 \cot^3 t}{\csc t} = 3 \cos t (\csc^2 t - 1) \)

   b. \( \tan x - \cot x = \sec x (2 \sin x - \csc x) \)

2. Find all solutions for:
   (a) \( 3 \tan x + \sqrt{3} = 0 \)
(b) \( \sec x - \cos x = 0 \)

(c) \( \csc^2 x - 3 = 0 \)

(d) \( 2 \sin^2 x = 7 \cos x + 5 \)

(e) \( 2 \cos 4x - 1 = 0 \)
(f) $4 \sin^2 \left( \frac{x}{2} \right) = 3$

(g) $80 \cos \left( \frac{\pi x}{3} + \frac{\pi}{4} \right) - 40\sqrt{2} = 0$

3. Find the exact value of $\sin \left( \frac{5\pi}{12} \right)$ if $\frac{5\pi}{12} = \frac{5\pi}{3} - \frac{5\pi}{4}$. 


4. Find the exact value of \( \cos(115^\circ) \cos(5^\circ) - \sin(115^\circ) \sin(5^\circ) \).

5. Find the exact value of \( \frac{\tan(\pi/15) + \tan(4\pi/15)}{1 - \tan(\pi/15) \tan(4\pi/15)} \).

6. Rewrite \( \sin(x) \cos(3x) + \sin(3x) \cos(x) \) as a single expression.
Review Problems

1. Convert $115^\circ$ to radians.

2. Convert $\frac{17\pi}{36}$ to degrees.

3. An arc created by a central angle of $\frac{2}{5}$ radians has a length of 80 ft, determine the radius of the circle.

4. Emmy chooses a horse that is 10 feet from the center of a merry-go-round. The merry-go-round makes $4\frac{1}{2}$ rotations per minute. Determine Jack’s angular velocity in radians per second. How far has Emmy travelled in 5 minutes?
5. Given \( \csc(\theta) = -\frac{7}{5} \) and \( \tan(\theta) < 0 \), find the value of \( \sec(\theta) \).

6. Given \( t \) corresponds to the point \( \left( \frac{1}{5}, -\frac{2\sqrt{6}}{5} \right) \) is on the unit circle, find the value of \( \sin(t) \), \( \sec(t) \), and \( \tan(t) \).

7. Given \( y = \frac{3}{2} \sin (5x - \pi) - 2 \), state the amplitude, period and phase shift of the graph. Sketch the graph.
8. State the domain and range of \( y = \tan(x) \)

9. State the domain and range of \( f(x) = \arcsin(x), g(x) = \arccos(x), \) and \( h(x) = \arctan(x) \).

10. From his hotel room window on the sixth floor, Saleh notices some window washers high above him on the hotel across the street. Curious as to their height above the ground, he quickly estimates the buildings are 50 ft apart and the angle of elevation to the workers is 80\(^\circ\). Leave all answers in exact form.
   a) How far apart are Saleh and the window washers?

   b) If Saleh’s hotel floor is 60ft above ground, how far are the window washers from the ground?
11. Simplify each composition, if possible.

\[
tan \left[ \arctan \left( -\frac{\sqrt{3}}{3} \right) \right] = \text{______________________________}
\]

\[
\arcsin \left[ \sin \left( \frac{5\pi}{6} \right) \right] = \text{______________________________}
\]

\[
\sin^{-1} [\cos(0)] = \text{______________________________}
\]

\[
tan \left[ \arcsin \left( \frac{\sqrt{x^2 - 25}}{x} \right) \right] = \text{______________________________}
\]

12. Find all solutions to \(\sqrt{3} \csc(x) + 3 = 1\)
13. Solve the equation $\sin(2x) + \sin(x) = 0$

14. Find all solutions to the equation $2\sin^2(3x) - 1 = 0$. 