Note #1 (Vectors, Dot Product.)

[Vectors]

(1) Find a vector $\overrightarrow{AB}$.
   
   (a) $A(-3, 4), B(1, -2)$

   (b) $A(0, 0), B(1, 1)$

   (c) $A(-2, 2), B(-1, 3)$
(2) If $\mathbf{a} = \langle -1, 2 \rangle$ and $\mathbf{b} = \langle 5, 3 \rangle$, find $|\mathbf{a}|$, $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, and $-3\mathbf{a} + 4\mathbf{b}$.

(3) If $|\mathbf{r}| = 2$, and $\mathbf{r}$ makes an angle of $210^\circ$ with the positive $x$-axis, find the component of the vector $\mathbf{r}$.

(4) If $\mathbf{a} = \langle 3, -4 \rangle$, find a vector with length 10 in the direction of $\mathbf{a}$. 
[Dot product]

(5) Find \( \mathbf{a} \cdot \mathbf{b} \).
  
(a) \(|\mathbf{a}| = 4, \ |\mathbf{b}| = 5\), the angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( \frac{\pi}{3} \)

(b) \( \mathbf{a} = \langle -2, -8 \rangle, \ \mathbf{b} = \langle 6, -4 \rangle \)

(c) \( \mathbf{a} = \mathbf{i} + \mathbf{j}, \ \mathbf{b} = \mathbf{i} - 2\mathbf{j} \)
(6) Find the angle between the vectors.
   
   (a) \( \mathbf{a} = \langle 6, 0 \rangle, \mathbf{b} = \langle 5, 3 \rangle \)

   (b) \( \mathbf{a} = \langle 3, 1 \rangle, \mathbf{b} = \langle 2, 4 \rangle \)
(7) Find the values of \( x \) such that the given vectors are orthogonal.

(a) \( \langle 4, x \rangle, \langle x, 1 \rangle \)

(b) \( \langle x, x \rangle, \langle 1, x \rangle \)
(8) A force \( \mathbf{F} = (-3, 4) \) is used to move an object from the point \((0, 2)\) to the point \((-3, 3)\). How much work is done by the force if distance is measured in meters and force is measured in Newtons?
(9) A boat heads in the direction $N30^\circ E$ with a speed of $40\text{ mph}$. The water current is flowing $S45^\circ E$ with a speed of $6\text{ mph}$. Find the true speed and direction of the boat.