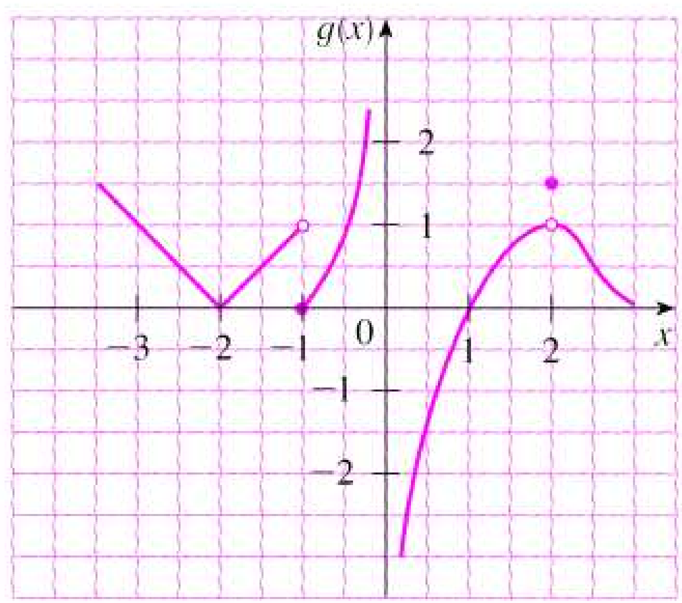
Note ♯3 (The Limit of a Function, Calculating Limits Using the Limit Laws, Continuity, Limits at Infinity)

[The Limit of a Function]

(1) State the value of the given quantity, if it exists, from the given graph.



a. $\lim_{x\to 0^-} g(x)$

b. $\lim_{x\to 0^+} g(x)$

c. $\lim_{x\to 0} g(x)$

d. $\lim_{x\to 2^-} g(x)$

e. $\lim_{x\to 2^+} g(x)$

f. $\lim_{x\to 2} g(x)$

g. $g(2)$

h. $\lim_{x\to -1^-} g(x)$

i. $\lim_{x\to -1^+} g(x)$

j. $\lim_{x\to -1} g(x)$

(2) Find the limit.

(a) $\lim_{x \to 3} \frac{1}{(x-3)^8}$

(b) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)}$

(c) $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)}$

(d) $\lim_{x \to -2^-} \frac{x-1}{x^2(x+2)}$

(e) $\lim_{x \to -2} \frac{x-1}{x^2(x+2)}$

[Calculating Limits Using the Limit Laws]

(3) Evaluate the limit.

(a) $\lim_{x\to -3} \dfrac{x^2 - x - 12}{x + 3}$

(b) $\lim_{x\to 1} \dfrac{x^2 - x - 2}{x + 1}$

(c) $\lim_{t\to 0} \dfrac{\sqrt{2-t} - \sqrt{2}}{t}$

(d) $\lim_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

(e) $\lim_{t \to 1} \left\langle 2t - 3,\ \dfrac{t^2 - t}{t - 1} \right\rangle$

(4) Find the limit.

(a) $\lim_{x\to -4^-} \frac{|x+4|}{x+4}$

(b) $\lim_{x\to -4^+} \frac{|x+4|}{x+4}$

(c) $\lim_{x\to -4} \frac{|x+4|}{x+4}$

(5) Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

evaluate each of the following limits if it exists.

(a) $\lim_{x \to 0^+} f(x)$

(b) $\lim_{x \to 0^-} f(x)$

(c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 1} f(x)$

(e) $\lim_{x \to 2^-} f(x)$

(f) $\lim_{x \to 2^+} f(x)$

(g) $\lim_{x \to 2} f(x)$

[**Continuity**]

(6) Explain why the function $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$ is discontinuous at $x = 0$.

(7) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

[Limits at Infinity]

(8) Evaluate the limit.

(a) $\lim\limits_{x\to\infty} \dfrac{-5}{x}$

(b) $\lim\limits_{x\to\infty} \dfrac{x+1}{x^2+3}$

(c) $\lim\limits_{x\to\infty} \dfrac{7x^3}{x^3-3x^2+3x}$

(d) $\lim\limits_{x\to-\infty} \dfrac{3x^7+5x^2-3}{6x^2-7x+8}$

(e) $\lim\limits_{x\to\infty} \dfrac{9x^4-2x}{2x^4+5x^2-8}$

(f) $\lim\limits_{x\to-\infty} \dfrac{9x^4-2x}{2x^4+5x^2-8}$

(g) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x+1}$

(h) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2+1}}{x+1}$

(i) $\lim\limits_{x\to\infty} \dfrac{x-3}{\sqrt{4x^2+21}}$

(j) $\lim\limits_{x\to-\infty} \dfrac{5-3x^3}{\sqrt{x^6+21}}$