Note ♯4 (Exam1 Review)

(1) Vector **a** starts at the point $(-3, 1)$ and ends at the point $(6, 3)$. Find a unit vector of **a**.

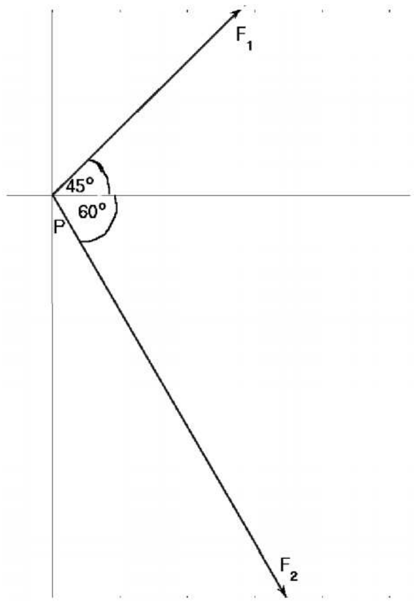
(2) Determine a vector equation of the straight line which passes through the point $(1, -1)$ and is parallel to the vector $\langle -2, 3 \rangle$.

(3) Find the angle $\angle ACB$ of the triangle with the vertices, $A(3, 0)$, $B(0, 3)$, $C(5, 4)$.

(4) Find the distance from the point $(1, 2)$ to the line $y = 3x - 4$.

(5) Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act on an object. The magnitude of $\mathbf{F_1}$ is 10 lb and it makes a 120° angle with the positive x-axis. The magnitude of $\mathbf{F_2}$ is 8 lb and it makes a 45° angle with the positive x-axis. Find the magnitude of the resultant force $\mathbf{F}$.

(6) Two force $\mathbf{F_1}$ and $\mathbf{F_2}$ with magnitudes $8N$ and $14N$, respectively, act on an object at a point P as shown in figure below. Find the resultant force and its magnitude.



(7) A constant force with the vector representation $\mathbf{F} = \langle 10, 18 \rangle$ moves an object along a straight line from the point $(2, 3)$ to the point $(4, 9)$. Find the work done, if the distance is measured in meters and the magnitude of the force is measured in newtons.

(8) Consider the curve $x(t) = t - 2$, $y(t) = t^2 - 3$. (a) Is the point $(4, 40)$ on the graph of the curve? (b) Eliminate the parameter to find a cartesian equation. (c) Sketch the curve.

(9) Consider the curve $x = 3 + \cos t$, $y(t) = -1 + \sin t$. (a) Eliminate the parameter to find a cartesian equation. (b) Sketch the curve.

(10) Find the equation of the line perpendicular to the vector $\langle 3,5\rangle$ and passing through the point $(5,1)$.

(11) Consider the line $x = 8 - 2t$, $y = 14 + 7t$. (a) Find a vector perpendicular to the line, (b) Find the Cartesian form and (c) sketch the graph.

(12) Consider the lines $\vec{r}(t) = \langle -4 + 2t, 5 + t \rangle$ and $\vec{s}(w) = \langle 2 + 3w, 4 - 6w \rangle$. Determine whether the lines are parallel, perpendicular or neither. If they are not parallel, find the intersection point.

(13) Simplify.

(a) $\tan(\arccos(\frac{1}{4}))$

(b) $\sin(\arctan(2))$

(c) $\tan(\arcsin(3x))$

(14) Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

evaluate each of the following limits if it exists.

(a) $\lim_{x \to 0^+} f(x)$

(b) $\lim_{x \to 0^-} f(x)$

(c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 2^-} f(x)$

(e) $\lim_{x \to 2^+} f(x)$

(f) $\lim_{x \to 2} f(x)$

(15) Evaluate the limit.

(a) $\lim_{x \to 2} \frac{x-1}{x^2+4}$

(b) $\lim_{x \to -1} \frac{x^2-x-2}{x+1}$

(c) $\lim_{t \to 0} \dfrac{\sqrt{2-t} - \sqrt{2}}{t}$

(d) $\lim_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

(e) $\lim_{x \to -4^-} \frac{|x+4|}{x+4}$

(f) $\lim_{x \to 4^-} \frac{|x-4|}{x^2-2x-8}$

(g) $\lim_{x \to -1^-} \dfrac{x-2}{x+1}$

(h) $\lim_{x \to -\infty} \dfrac{-x^3+2x^2-4x}{8+4x^2-5x^3}$

(i) $\lim_{x \to \infty} \dfrac{e^x+2e^{-x}}{2e^x-e^{-x}}$

(j) $\lim_{x \to -\infty} \dfrac{e^x+2e^{-x}}{2e^x-e^{-x}}$

(k) $\lim_{x \to 3^+} e^{3/(x-3)}$

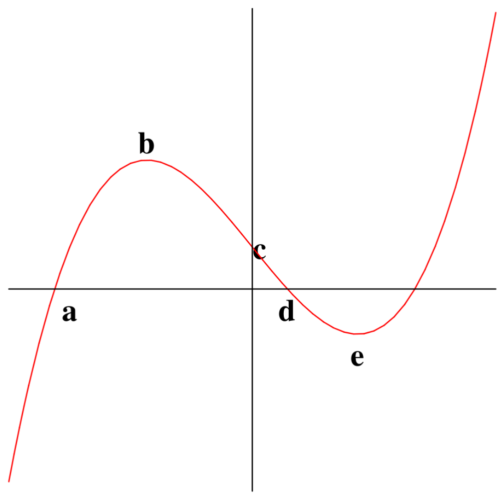
(l) $\lim_{x \to 3^-} e^{3/(x-3)}$

(m) $\lim_{x \to -\infty} \dfrac{\sqrt{1+4x^6}}{5-2x^3}$

(16) Given $-2x - 2 \leq f(x) \leq \frac{1}{2}x^2$, compute $\lim_{x \to -2} f(x)$.

(17) Find the vertical and horizontal asymptotes of $f(x) = \frac{(x-1)(x+3)}{x^2 - 1}$.

(18) Given the graph of $f(x)$ below, sketch the graph of the derivative.



(19) Given $f(x) = \begin{cases} x - 4a & \text{if } x < -2 \\ ax^2 & \text{if } x \geq -2 \end{cases}$. Find the value of a which makes the function continuous everywhere.

(20) Which of following intervals must contain a solution to the equation $2x^3 + 16x + 3 = 22$?

(a) $[-2, -1]$
(b) $[-1, 0]$
(c) $[0, 1]$
(d) $[1, 2]$
(e) $[2, 3]$

(21) Find the average rate of change of $f(x) = x^2 + 6$ from $x = -3$ to $x = 1$.

(22) Fine $f'(x)$ using the limit definition of the derivative for $f(x) = 3x^2 - 4$.

(23) Using the result above, find the equation of the tangent line to the graph of $f(x) = 3x^2 - 4$ at $x = 2$.

(24) Find $f'(x)$ using the limit definition of the derivative for $f(x) = \sqrt{3x+1}$.

(25) Using the result above, find the equation of the tangent line to the graph of $f(x) = \sqrt{3x+1}$ at $x = 1$.

(26) Find $f'(x)$ using the limit definition of the derivative for $f(x) = \dfrac{-2}{x+2}$.

(27) Using the result above, find the equation of the tangent line to the graph of $f(x) = \dfrac{-2}{x+2}$ at $x = 0$.

