NOTE §5 (DERIVATIVES OF POLYNOMIAL AND EXPONENTIAL FUNCTIONS, THE PRODUCT AND QUOTIENT RULES, DERIVATIVES OF TRIGONOMETRIC FUNCTIONS, AND CHAIN RULE, IMPLICIT DIFFERENTIATION)

[Derivatives of Polynomial and Exponential Functions, The Product and Quotient Rules, Derivatives of Trigonometric Functions, and Chain Rule]

(1) Differentiate the functions.
   (a) \( f(x) = \frac{7}{4}x^4 - 3x^2 + 12 \)

   (b) \( H(u) = (3u - 1)(u + 5) \)

   (c) \( F(r) = \frac{8}{r^3} \)
(d) \( y = \frac{\sqrt[3]{x} + x}{x^2} \)

(e) \( k(x) = e^x + x^e \)

(f) \( f(x) = (3x^2 - x)e^x \)
(g) \( y = \frac{e^x}{4 - e^x} \)

(h) \( G(x) = \frac{x^2 - 3}{5x + 1} \)

(i) \( F(y) = \left( \frac{1}{y^2} - \frac{3}{y^4} \right) (y + 2y^3) \)
(j) $V(t) = \frac{7 + t}{te^t}$

(k) $F(x) = (1 + x + x^2)^{200}$

(l) $g(\theta) = \cos^2 \theta$
(m) \( y = e^{\tan \theta} \)

(n) \( F(t) = e^{t \sin 2t} \)

(o) \( f(t) = \tan(\sec(\cos t)) \)
(2) Find $f'(x)$ and $f''(x)$.

(a) $f(x) = (x^3 + 1)e^x$

(b) $y = e^{e^x}$
(3) Let \( r(x) = f(g(h(x))) \), where \( h(1) = 2, g(2) = 3, h'(1) = 4, g'(2) = 5, \) and \( f'(3) = 6 \). Find \( r'(1) \).

(4) If \( g(x) = f(3f(4f(x))) \), where \( f(0) = 0 \) and \( f'(0) = 2 \), find \( g'(0) \).
(5) Find the 2020th derivative of $y = \cos 2x$. 

(6) Find the 2020th derivative of $f(x) = xe^{-x}$. 
(7) If $f$ and $g$ are the functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$. Find $u'(1)$, $v'(1)$, and $w'(1)$. 
[Implicit Differentiation]

(8) Find \( \frac{dy}{dx} \).

(a) \( x^3 - xy^2 + y^3 = 1 \)

(b) \( \cos(xy) = 1 + \sin y \)
(c) \( e^y \sin x = x + xy \)

(d) \( x \sin y + y \sin x = 1 \)

(9) If \( g(x) + x \sin(g(x)) = x^2 \), find \( g'(x) \).
(10) Find an equation of the tangent line to the curve at the given point.

(a) $x^2 + 2xy + 4y^2 = 12, \ (2, 1)$

(b) $y \sin 2x = x \cos 2y, \ (\pi/2, \pi/4)$