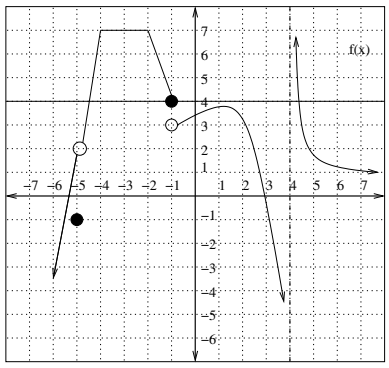
Fall 2021 Math 151

Week in Review 3

courtesy: Amy Austin

(covering sections 2.2, 2.3, 2.5)

1. Use the graph of $f(x)$ below to compute the following limits, or explain why the limit does not exist.



a) $\lim_{x\to -1^-} f(x)$ b) $\lim_{x\to -1^+} f(x)$

c) $\lim_{x\to -1} f(x)$ d) $\lim_{x\to -5} f(x)$

e) $\lim_{x\to 4^+} f(x)$ f.) $\lim_{x\to -2} f(x)$

2. Find $\lim_{x\to -2} \dfrac{x-1}{x+2}$ or explain why it does not exist.

3. Find $\lim_{x\to 3} \dfrac{x-5}{x^2-9}$ or explain why it does not exist.

4. Find $\lim_{x\to 4} \dfrac{x-1}{(x-4)^2}$ or explain why it does not exist.

5. Compute the exact value of the following limits. If the limit does not exist, support your answer by evaluating left and right hand limits.

   (a) $\lim_{x\to 1}(4x^3-3x+1)$

   (b) $\lim_{x\to -5} \dfrac{x^2+5x}{x+5}$

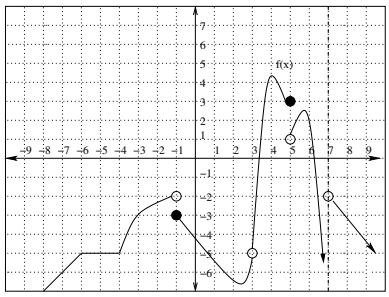
   (c) $\lim_{x\to 2} \dfrac{x-\sqrt{3x-2}}{x^2-4}$

   (d) $\lim_{h\to 0} \dfrac{(3+h)^{-1}-3^{-1}}{h}$

   (e) $\lim_{x\to 2} \dfrac{x^2-4}{|\,x-2\,|}$

   (f) $\lim_{x\to 1} f(x)$ if it is known that $4x \le f(x) \le x+3$ for all x in $[0,2]$.

6. Refer to the graph of $f(x)$ below. Find all values of x where $f(x)$ is discontinuous. For these values of x, is $f(x)$ it continuous from the right, left or neither? Support your answer.



7. Determine whether the following functions are continuous at the indicated value of x. Support your answer.

   (a) $f(x) = \dfrac{x^2-4}{x-2}$, $x = 2$.

   (b) $f(x) = \dfrac{1}{x-1}$, $x = 1$

   (c) $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x+4 & \text{if } x \ge 1 \end{cases}$, $x = 1$

8. Suppose it is known that $f(x)$ is a continuous function defined on the interval $[1,5]$. Suppose further it is given that $f(1) = -3$ and $f(5) = 6$. Give a graphical arguement that there is at least one solution to the equation $f(x) = 1$.

9. If $g(x) = x^5 - 2x^3 + x^2 + 2$, use the Intermediate Value Theorem to find an interval which contains a root of $g(x)$, that is contains a solution to the equation $g(x) = 0$.

10. Find the values of c and d that will make

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \le x \le 2 \\ 4x & \text{if } x > 2 \end{cases}$$

continuous on all real numbers. Once the value of c and d is found, find $\lim_{x\to 1} f(x)$ and $\lim_{x\to 2} f(x)$.