



## NOTE #3 (VOLUMES BY CYLINDRICAL SHELLS, WORK)

### [Volumes by Cylindrical Shells]

- (1) Find the volume of the solid obtained by rotating the region bounded by  $y = \sqrt{x-1}$ ,  $y = 1$ ,  $y = 3$  and the  $y$ -axis around the  $x$ -axis.



- (2) Find the volume of the solid obtained by rotating the region bounded by  $y = 3x - x^2$  and  $y = 3x - 9$  around the line  $x = 5$ .



- (3) Find the volume of the solid obtained by rotating the region bounded by  $x = y^2 - 3y + 4$  and  $x = -y^2 + y + 4$  around the line  $y = -1$ .



- (4) Find the volume of the solid obtained by rotating the region bounded by  $y = x^2$  and  $x = y^3$  around the line  $x = -3$ .



- (5) Find the volume of the solid obtained by rotating the region bounded by  $y = x^2 - 6$  and the line  $y = -2$  around the line  $x$ -axis.



[Work]

- (6) A particle is moved along the  $x$ -axis by a force that measures  $f(x) = x^3\sqrt{x^2 - 9}$  pounds at a point  $x$  feet from the origin. Find the work done in moving the particle from  $x = 3$  to  $x = 5$ .



- (7) Suppose a spring has a natural length of  $10\text{ cm}$  and it takes  $8\text{ N}$  of force to hold it stretched to  $30\text{ cm}$ . How much work is required to stretch it from  $15\text{ cm}$  to  $25\text{ cm}$ ?



- (8) Suppose it takes  $10 \text{ ft} - lb$  to stretch a spring from  $2 \text{ ft}$  to  $5 \text{ ft}$  beyond its natural length.
- (a) How much work is require to stretch the spring from  $1 \text{ ft}$  to  $4 \text{ ft}$  beyond its natural length?
  - (b) How far beyond its natural length would a force of  $2 \text{ lb}$  keep the spring stretched?





- (9) A rope that is  $30\text{ m}$  long and weighs  $10\text{ kg}$  hangs over the edge of the building.
- (a) How much work is done pulling all the rope to the top?
  - (b) How much work is done pulling half the rope to the top?
  - (c) Is the work to pull the all the rope to the top double pulling half the rope to the top? Why or why not?



- (10) A 400 *lb* piano is lifted 36 *ft* to be placed into an apartment through a large window. The cable used to lift the piano weighs 1 *lb* for every 6 *ft*. How much work is done?



- (11) A swimming pool is  $10\text{ m}$  wide and  $20\text{ m}$  long. The pool is  $7\text{ m}$  deep at one end and slopes up the surface at the other end.
- (a) If the pool is full, how much work is done pumping all the water out of the pool?
  - (b) If the pool is full, how much work is done if the water is pumped out until  $3\text{ m}$  of water is left in the deep end? Setup the integral only.
  - (c) If the pool was only filled till it is  $5\text{ m}$  deep in the deep end, how much work is done to pump all the water out of the pool? Setup the integral only.



- (12) A tank is the shape of a inverted frustum of a cone (an inverted cone with the tip cut off). The radius of the bottom of the tank is  $3 m$ , and the radius of the top of the tank is  $6 m$ . The height of the tank is  $6 m$  and the spout at the top is  $2 m$  high.
- (a) If the tank is full, how much work is done pumping all the water out of the tank?
  - (b) If the tank is filled  $4 m$  deep, how much work is done pumping all the water out of the tank? Setup the integral only.